Minimal Observability for Transactional Hierarchical Services

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Abstract

For complex services, logging is an integral part of many middleware aspects, especially, transactions and monitoring. In the event of a failure, the log allows us to deduce the cause of failure (diagnosis), recover by compensating the logged actions (atomicity), etc. However, for heterogeneous services, logging all the actions is often impracticable due to privacy/security constraints. Also, logging is expensive in terms of both time and space. Thus, we are interested in determining the absolute minimal number of actions that needs to be logged, to know with certainty the actual sequence of executed actions from any given partial log. This problem happens to be NP-Complete. We consider complex services represented as a hierarchy of services, and propose a decomposition mechanism which dramatically decreases the complexity (up to 2 exponentials). The decomposition also works for distributed services.

1 Introduction

An interesting problem for complex systems is to determine a minimal set of actions that needs to be observable such that a given property holds. Some of the properties studied in literature of discrete event systems are normality [8], observability [7], state observability [11], diagnosability [15], etc. Our system corresponds to a composite (workflow) Web service. A Web service [1] refers to an online service accessible via Internet standard protocols. A composite service, composed of already existing (component) services, combines the capabilities of its components to provide a new service. A service schema which specifies the execution order of its components, can be modeled as a graph, performing actions on global variables. We do not tackle here the modelization of a service as a graph, which should be handled with care to yield a graph of reasonable size (see [17] and example 1).

Our long-term objective is to provide a transactional framework for (composite) Web services. A transaction can be considered as a group of actions encapsulated by the operations Begin and Commit/Abort, having the following properties: Atomicity (A), Consistency (C), Isolation (I) and Durability (D). Here, we focus on the atomicity aspect, that is, either all the actions of a transaction are executed or none. In the event of a failure, atomicity is preserved by compensation [4, 16]. Compensation consists of executing the compensating actions, corresponding to each executed action of the failed process, in reverse order of the original execution. Many advanced transactional models have also been proposed, e.g. “semantic compensation” [16] which do not require any knowledge of the execution sequence. However, their application to more autonomous systems like Web Services has been limited, where the default compensation mechanism of the widely used Business Process Execution Language (BPEL) is to “execute the completed actions in reverse order”. Thus, for compensation to be feasible, we need to reconstruct each executed action or the complete history of any execution. To achieve that, we maintain a log of the observable actions. In addition to the obvious space overhead of logging, the complete log may not always be accessible. For a composite service, the providers of its component services are different. As such, their privacy/security constraints may prevent them from exposing (part of) the logs corresponding to the execution at their sites. Also, heterogeneity may lead to the logs being maintained in different formats, rendering some of them incomprehensible. Existing Web services specifications to provide transactional guarantees, such as WS-Coordination, WS-AtomicTransaction and WS-BusinessActivity [18], are basically distributed agreement protocols which are based on the assumption that “all state transitions are reliably recorded” and can be compensated. Our approach is targeted towards a more heterogeneous environment where all transitions may not be observable. Hence, we want from a partial log of the observable actions to know with certainty the actual sequence of executed actions, to be able to compensate it.

Section 2 introduces the required formal preliminaries including the precise problem statement. Clearly, we are interested in logging the smallest number of actions possible. However, determining the minimal number of action-
s to log, such that any execution of a system is compensable, is NP-Complete. This is not very surprising, since the closely related sensor selection problems [19, 10] are also NP-Complete (see section 3). Also, the problem cannot be approximated [13] in polynomial time, which means that polynomial time algorithms cannot give a minimal set for all graphs, and that for many graphs, the observable set produced would be much bigger than the minimal set.

A complex service is often constructed hierarchically (see section 4), with some services at a high level corresponding to many composite services at a lower level. Each hierarchical level potentially describes the interactions at different levels of abstraction, e.g., the top level may describe the interactions between several providers, then the next level between services of a provider, and so on. Moreover, components can be reused in a hierarchical system, giving a compressed way to represent big systems. Hierarchical systems are often used for words [12], Finite State Machines [2], and even trees [9]. For words, e.g., hierarchical structures correspond to the LZ compression [12]. We show in section 5 how to use the hierarchical representation to compute efficiently a minimal observable set of transitions. The algorithm is not straightforward since the log of both minimal sets of actions of different components is not necessarily enough to recover the actual sequence of executed actions of the whole graph. One solution could be to resort to function summarization, but then only an overapproximation of the minimal set of actions would be obtained. Nevertheless, we show that it suffices to run the algorithm with slightly different parameters on each component. We thus obtain a divide and conquer algorithm. We present a theoretical complexity analysis which illustrates the benefit of our method (up to two exponential better when using the full hierarchical representation and one exponential better by using the hierarchical representation even if components are used only once, compared to flattening the hierarchical graph), that is verified experimentally (section 7).

2 Preliminaries

Formally, we model a transactional service as a 4-tuple $M = (Q, s_0, s_f, T)$, where $(Q, T)$ is a graph ($q \in Q$ is called a state and $t \in T$ a transition) and $s_0 \in Q$ and $s_f \in Q$ are the initial and final states, respectively.

Our systems are thus graphs with a unique input and output point, each node and arc corresponds to a state and transition, but we ignore the alphabet. We assume that the service $M$ does not have any unreachable states and that all states can reach the final state $s_f$. For convenience, we also assume that there are no outgoing edges from $s_f$ and no incoming edges to $s_0$. We say that an execution sequence $\rho = \tau_1 \cdots \tau_n \in T^*$ is a path of $M$ if there exists $q_0, \ldots, q_n \in Q^{n+1}$ with $\tau_i = (q_{i-1}, q_i)$ for all $1 \leq i \leq n$. A path is called initial if furthermore $q_0 = s_0$. We denote by $P(M)$ the set of initial paths in $M$. Finally, we denote by $|M|$ the size of $M$, that is, its number of transitions.

In general, for any execution $\rho$, we call observation projections the observation we have after $\rho$ was executed (a sequence of actions, control points, data . . .). We say that an observable projection $\sigma$ is uncertain if there exists two paths having the same projection. The service $M$ is execution sequence detectable iff none of its observable projections are uncertain.

Definition 1 Let $T_O \subseteq T$ be the set of observable transitions. The observation projection $\text{Obs}_O : T^* \rightarrow T_O^*$ is the morphism with $\text{Obs}_O(a_1 \ldots a_n) = o_1 \ldots o_n$ with $o_i = a_i$ if $a_i \in T_O$, and $o_i = \varepsilon$ if $a_i \in T \setminus T_O$, with $\varepsilon$ the empty word.

That is, $\text{Obs}_O(\rho)$ is the subsequence of $\rho$ obtained by eliminating from $\rho$ every occurrence of a tuple which is not in $T_O$. With such an observation projection $\text{Obs}_O$, the only way of having execution sequence detectability is to have every transition observable. Indeed, as soon as there exists even one non-observable transition, the service is not execution sequence detectable. Else, let us take a path $\rho \tau$ with the last transition $\tau \notin T_O$. Then, $\text{Obs}_O(\rho \tau) = \text{Obs}_O(\rho)$. A usual way to overcome such a problem is to ask for certainty only up to the last few events of the sequence [11]. However, this workaround does not make sense in our framework since if we cannot compensate the very last action, then we cannot compensate any action at all. As such, we design a new observation mechanism, where the last control point reached before failure is monitored, even if the last action is not logged. In practice, it means that every state that is reached is monitored, and overwrite the previous state in a special memory buffer.

Definition 2 Let $M$ be a service, $T_O \subseteq T$. The observation projection $\text{Obs}^{\text{last}}_O : T^* \rightarrow (T_O, Q)$ is the function $\text{Obs}^{\text{last}}_O(\rho) = (\text{Obs}_O(\rho), q)$ for all $\rho \in P(M)$ ending in $q$.

We will stick with this definition of observability for the rest of the paper. As mentioned before, we are interested in logging as few transitions as possible.

Problem statement. Given an service $M = (Q, s_0, s_f, T)$, we call $T_O$ an observable set of transitions if the service is execution sequence detectable with $\text{Obs}^{\text{last}}_O$. We want to determine a minimal observable set of transitions $T_O \subseteq T$.

The cardinality of such a minimal observable set $T_O$ of a service $M$ is referred to as its observable size $MO(M) = |T_O|$. Notice that as is usual with decision and computation algorithms, it is sufficient to have an algorithm which from a service gives its observable size. That is, we can derive a
minimal observable set of the service based on knowledge of its observable size in polynomial time.

**Example 1** We consider in Fig. 1 a travel funds request service, inspired by the workflow in [14]. It involves different departments across organizations, and it is hierarchical in that the delivery cheque service is hierarchically described.

We model the service using the service $M = (S, s_0, s_f, T)$, as shown in Fig. 2. Notice that this service is a simplification, since for instance the choice between the team leader or supervisor approvals is not represented. The reason is that they are both associated with an empty compensating transition, hence knowing which path was taken here is not necessary to be able to perform recovery. However, it is necessary to know which bank issued the cheque in order to be able to compensate it, by a “Cancel Last American Express (Citibank) Cheque”. Note that we do not exclude the logging of data values (in some persistent storage) required for compensation. For instance, if there wasn’t any “Cancel Last Cheque” mechanism, then it would be needed to log the amount and account number associated with the “Update Accounts Database” transition. Recovery would manually credit the amount of money written in the log to the corresponding account. Obviously, we cannot save on logging the data values, but we optimize the logging associated with the path visited. Our experiments performed on BPEL representations of some workflows reveal that one transition out of five is logged (which is confirmed in section 7) and that data values logs are small compared to logging the path.

Now, let $T_O = \{e_2, e_0, e_9\}$ and a failure occurs while processing $e_0$, that is, the cheque is not issued or delivered correctly. Then, $\text{Obs}_{last}(e_1 e_2 e_0 e_9) = (e_2, s_0) = \text{Obs}_{last}(e_1 e_2 e_0 e_9)$. Thus, we do not know if an American Express or Citibank cheque was processed. With $T_O = \{e_2, e_0, e_9\}$, we have $\text{Obs}_{last}(e_1 e_2 e_0 e_9) = (e_2, s_0) \neq \text{Obs}_{last}(e_1 e_2 e_0 e_9) = (e_2 e_0, s_0) \neq \text{Obs}_{last}(e_1 e_2 e_0 e_9) = (e_2 e_0, s_0)$, and $T_O'$ is an observable set of transitions. Every path from $s_0$ to $s_f$ uses at least one transition from $T_O'$.

**Figure 2. Modelization of Fig. 1.**

### 3 Problem Hardness

We first relate the problem of computing $MO(M)$ using our definition of observable projections with other known problems. We state now that computing the minimal observable set is equivalent to the unconnected subgraph problem, also called the minimal marker placement problem [10], in the meaning of the following proposition.

**Proposition 1** Let $M$ be a service and $T_O$ a subset of transitions of $M$. Denote by $M'$ the service $M$ obtained by deleting all transitions belonging to $T_O$. Then, $T_O$ is an observable set of $M$ if and only if there does not exist a pair of paths $p_1 \neq p_2$ of $M'$ with $p_1$ beginning and ending at the same states as $p_2$.

To prove proposition 1, it suffices to prove that if there does not exist a pair of paths $p_1 \neq p_2$ of $M'$ with $p_1$ beginning and ending at the same states as $p_2$, then any observable projection $(\sigma, q_{n+1})$, we can reconstruct in a unique way a path with $\text{Obs}_{last}(\rho) = (\sigma, q_{n+1})$. The converse is trivial. Indeed, it suffices to define the only path $p_1$ of $M'$ between $q_i$ and $q_{i+1}$ for $\sigma = (q_1, q_i)_{i=0}^{n-1}$, and $i = 0 \cdots n$ (we fix $q_0 = s_0$ the initial state of $M'$, and recall that $q_{n+1}$ is the last observed state). Then, the path $\rho = \rho_1(q_1, q_i)\rho_2 \cdots (q_n, q_0)\rho_n$ is the only path with $\pi_{last}(\rho) = (\sigma, q_{n+1})$. The search for each path $\rho_i$ can be made in linear time by a simple depth first search in $M'$.

The fact is that the marker placement problem is an NP-Complete problem. The question is then to know if there is a structural subclass of graphs which has a tractable algorithm to give the minimal observable size. We know from [10] that the minimal marker placement problem is NP-Complete even for acyclic graphs. However, the proof uses a graph with unbounded (in and out) degree. We show that the problem is NP-Complete even if the graph is both acyclic and the sum of its in and outdegree bounded by 3 (that is, indegree 2 and outdegree 1, or vice versa). The core of the proof follows the same strategy as [10], but the
encoding to get a unique starting and ending point is both easier to understand and allows a lower in and outdegree.

**Theorem 1** Let $M$ be a service, and $k$ a number. Knowing whether $MO(M) \leq k$ is NP-Complete, even if the corresponding graph is acyclic and the sum of in and outdegree of every node bounded by 3.

**Proof.** Let $M$ be a system. We reduce Vertex Cover to the problem of finding a subset of transitions $T_O$ of $M$, such that, there are no two paths $\rho_1 \neq \rho_2$ beginning and ending at the same nodes, and not using transitions of $T_O$. Let us take an undirected graph $(V, E)$ and a number $k$. We want to know whether there is a subset $V_O$ of $V$ of size $\leq k$ such that for all $(v, w) \in E$, at least one of $v, w$ belongs to $V_O$. This problem is NP-complete even with $(V, E)$ of degree 3. The first FSM $M$ we build has a state space $S = V_1 \cup V_2 \cup E_1 \cup E_2$ where $V_1 = \{v_i \mid v \in V\}$ and $E_2 = \{e_i \mid e \in E\}$. Furthermore, for $v, w \in V$ and $e \in E$, we have transitions

1. $(e_1, v_1) \in T$ iff $v \in e$ iff $(v_2, e_2) \in T$
2. $(v_1, w_2) \in T$ iff $v = w$.

A graphical representation of $M$ appears in Fig. 3.

Assume that there is a subset $V_O$ of $V$ of size $k$ such that for all $(v, w) \in E$, at least one of $v, w$ belongs to $V_O$. Then, defining $T_O = \{(v_1, v_2) \mid v \in V_O\}$, we have that there are no two paths $\rho_1 \neq \rho_2$ with $\rho_1$ and $\rho_2$ beginning and ending at the same nodes, and not using transitions of $T_O$. By contradiction, else we would have $\rho_1, \rho_2$ both from some $e_1 \in E_1$ to some $e_2 \in E_2$ and not using transitions of $T_O$. By definition of $T_O$, it means that for $\rho_1$, there exists a node $v \in e, v \in f$ such that $v \notin V_O$. Similarly, for $\rho_2$ with a node $w$. Since $\rho_1 \neq \rho_2$, we have that $v \neq w$; hence $e = (v, w)$ contradicts $V_O$ is a vertex cover.

Conversely, assume that there is a set of transitions $T_O$ of size $k$ such that there do not exist two paths from and to the same state without using $T_O$. We build the set of nodes $V_O = \{v \mid (v_1, v_2) \in T_O\} \cup \{v \mid \exists e_i (e_i, v_1) \in T_O\} \cup \{v \mid \exists e_i (v_2, e_2) \in T_O\}$. Clearly, $|V_O| \leq |T_O| = k$. We prove now that $V$ is a vertex cover of $(V, E)$. Assume by contradiction that there exists an edge $e = (v, w)$ such that $v, w \notin V_O$. Then, we argue that $e_1, v_1, v_2, e_2$ and $e_1, w_1, w_2, e_2$ are two paths not using $T_O$, a contradiction.

However, so far, the graph defined is not a system since it has several states with indegree 0 (the $e_1 \in E$), and several states with outdegree 0 (the $e_2 \in E$). Moreover, the indegree of states $(v_1)_{v \in V}$ and the outdegree of states $(v_2)_{v \in V}$ can be 3 (the degree of the undirected graph $(V, E)$). However, it is acyclic. For the degree, one can safely transform any node $v_1$ with 3 ingoing transitions from $e_1, f_1, g_1$ by having two nodes $v_1, v'_1$ with transitions $(e_1, v'_1), (f_1, v'_1), (v'_1, v_1)$ and $(g_1, v_1)$. Hence all nodes have indegree at most 2. The same can be done for outdegree. The size of the minimal observable set of transitions will not change with such a transformation. Actually, with such a technique, we could start from an undirected graph of any degree.

Making the graph a system is a little more involved. We use the graph $G$ from Fig. 2. It then suffices to create a balanced binary tree of transitions with root $s_1$ such that there are $E$ leaves. This tree has $O(2E)$ nodes, that we add to the system $S$ we built from $(V, E)$. The root of the tree is the unique initial node, and every leaf is connected to a node $(e_2)_{e \in E}$ through a copy of graph $G$. The same is done for nodes $(e_2)_{e \in E}$ connected through copies of $G$ to a balanced binary tree with root $s_f$ the unique final state. This system has $O(|V| + |E|)$ nodes, is acyclic and of total degree 3. Now, it is easy to show that if the minimal vertex cover has $k$ vertices, then the minimal observable set of transitions is of size $k + |E|$. Indeed, there are $2|E|$ copies of the graph $G$ each of which requires 2 observed transitions. Once these transitions are observed, the two balanced trees are totally disconnected from each other and from the first system we had built (since every path from the initial to the final state of the graph $G$ uses one of the two observed transitions), and hence we need to observe exactly $k$ more transitions. Notice that connecting directly the tree with $S$ without using $G$ would not work since it would potentially connect $s^0, s^l$ through two different paths $s^0 \rightarrow e_1 \rightarrow e_2 \rightarrow s^l$ and $s^0 \rightarrow f_1 \rightarrow f_2 \rightarrow s^l$, with $e, f \in E$.

This theorem does not mean that the problem is impossible to solve, but that it cannot be solved for all possible services. For instance, the complexity of the brute force method which generates every subset of transitions and tests whether it is observable, is $O(2^{|M|})$ for a service $M$ with $|M|$ transitions. The question then is which structural property makes the problem easier to solve and often holds for (real life) composite services. We propose hierarchical services as a candidate property.

**4 Hierarchical Services**

Hierarchical services provide an efficient way to model large and complex services by allowing a modular decomposition. We consider hierarchical services where two transitions (supertransitions) can be further
refined into another service. A hierarchical service $H$ is a finite sequence $\langle M_i \rangle_{i=1, \ldots, n}$, where $M^i = (Q^i, s^i_0, s^i_f, T^i, (\tau^i_1, k^i_1), (\tau^i_2, k^i_2))$ is defined as follows:

- $(Q^i, T^i)$ is a finite graph,
- $s^i_0$ and $s^i_f$ are the initial and final states, respectively,
- $\tau^i_1, \tau^i_2 \in T^i \cup \{e\}$ are two supertransitions representing services $M^{k^i_1}, M^{k^i_2}$ respectively, with $k^i_1, k^i_2 > i$.

For instance, the workflow in Fig. 1 can be described by a hierarchical service $\langle M_1, M_2 \rangle$, where $M_2$ is made of a initial and final state, and two transitions $e_8, e_9$ from the initial to the final state. The service $M_1$ is very similar to Fig. 2, except that there is a unique transition $e_{10}$ between $s_0$ and $s_f$ instead of two. This is a supertransition $(\tau^1_1, k^1_1)$, with $\tau^1_1 = e_{10}$ and $k^1_1 = 2$, meaning that $e_{10}$ represents $M_2$.

With each hierarchical service $H$, we associate an ordinary service $\mathcal{H}$ obtained by taking $M^i$, and recursively substituting each supertransition by the service it represents. For example 1, $\mathcal{H}_1$ is depicted in Fig. 2. Given a hierarchical service $\langle \mathcal{H}_n \rangle$, $\mathcal{H}_j$ is a component of $\mathcal{H}_i$, if there is a supertransition $(t, j)$ in $\mathcal{H}_i$. We define the size $|H|$ of a hierarchical service $H$ as the sum of the number of transitions of its components $M^i$. Its diameter $||H||$ is the number of transitions of $\mathcal{H}$. The diameter $||H||$ of $H$ can be exponential in the size of $H$, because components can be reused several times (for instance, a supertransition of $H_2$ and two supertransitions of $H_4$ can represent $H_{10}$, in which case one does not need to redefine $H_{10}$ three times).

Now, let us define a hierarchical system $H$ with two levels. The top level $H_1$ has two states, one initial and one final, with two transitions $\tau_1, \tau_2$ from the initial to the final state. Transition $\tau_2$ is a supertransition. It is not easy to determine a minimal set of transitions for $H$. Consider first that $\tau_2$ describes a system $H_2$ similar to $H_1$, that is $t$-two transitions $\tau_3, \tau_4$ from the initial to the final state, but without supertransitions. The set $T_2 = \{\tau_3\}$ is a minimal observable set of transitions for $H_2$. Now, looking at $H_1$ as a normal system (without supertransitions), $T_1 = \{\tau_1\}$ is also a minimal observable set of transitions for $H_1$. We have furthermore that $T_1 \cup T_2$ is a minimal observable set of transitions for $H$.

However, if we take $H_2$ to be the system described in Fig. 2 and the associated minimal observable set $T_2' = T_2' = \{\tau_2, \\tau_3, \\tau_4\}$ of transitions described in example 1, then $T_1 \cup T_2'$ is not minimal among the observable set of transitions for $H$. The reason is that $T_2'$ is already an observable set of transitions, because all paths that pass through $H_2$ use at least one transition in $T_2'$, so they can be differentiated from the path $\tau_1$. That is, the fact that a subset of transitions is a minimal observable set of transitions is global to the whole graph, not local.

5 Algorithm for Minimal Observability

We turn now to defining an algorithm which uses the hierarchical structure of a complex service to compute the minimal observable set. First, we need the following notations. Given $\mathcal{T}_O$, a path $p$ is said to be an unobserved path if it does not use any transition of $\mathcal{T}_O$. For a service $M$ and a set of transitions $\mathcal{T}_O$ of $M$, we define the following predicates:

- $P_0(M, \mathcal{T}_O)$ holds if there does not exist more than one unobserved path between any two states $s_1 \neq s_2 \in Q$ ($\mathcal{T}_O$ is an observable set of transitions).
- $P_1(M, \mathcal{T}_O)$ holds if (i) $P_0(M, \mathcal{T}_O)$ holds, and (ii) there does not exist an unobserved path from $s_0$ to $s_f$.
- $P_1(M, \mathcal{T}_O)$ holds if (i) $P_0(M, \mathcal{T}_O)$ holds, and (ii) there do not exist states $s_1, s_2 \in Q$ such that (a) there is an unobserved path from $s_0$ to $s_2$, (b) there is an unobserved path from $s_1$ to $s_f$, and (c) there is an unobserved path from $s_1$ to $s_2$. We refer to such a combination of nodes and edges as an unobserved reverse cyclic pattern between $s_1$ and $s_2$ (within $M$).

For instance, on Fig. 2 with $\mathcal{T}'_O = \{e_2, e_6, e_9\}$, $P_0(\mathcal{T}'_O)$ holds because $\mathcal{T}'_O$ is observable, $P_1(\mathcal{T}'_O)$ holds because every path from $s_0$ to $s_f$ uses at least one transition of $\mathcal{T}'_O$, but $P_1(\mathcal{T}'_O)$ does not hold since there exists three non observable paths: $e_4$ from $s_2$ to $s_3$, $e_3$ from $s_0$ to $s_2$, $e_8$ from $s_2$ to $s_f$.

By definition, $P_1(M, \mathcal{T}_O) \Rightarrow P_1(M, \mathcal{T}_O) \Rightarrow P_0(M, \mathcal{T}_O)$, since for all $s$, there always exists a path from $s$ to $s$. Let $\epsilon < 0 < 1 < Y$. We define $\text{Best}(M, \mathcal{T}_O) = x \in \{e, 0, 1, 1'\}$ such that $P_2(M, \mathcal{T}_O)$ holds, but not $P_2(M, \mathcal{T}_O)$ with $xx > x$, with the convention $P_2(M, \mathcal{T}_O)$ is always true. Informally, Best refers to the best properties a given set of transitions can ensure, if observed.

Proposition 2 Let $C$ be a component of $M$, and $T_1, T_2$ be subsets of transitions of $C$, respectively such that $\text{Best}(C, T_1) = \text{Best}(C, T_2)$. Then, for all subset of transitions $\mathcal{T}_O$ of $M \setminus C$, we have $\text{Best}(M, \mathcal{T}_O \cup T_1) = \text{Best}(M, \mathcal{T}_O \cup T_2)$.

Proof. Let $C = (Q', s'_0, s'_f, T')$, and $\text{Best}(M, \mathcal{T}_O \cup T_1) = x$. Let us assume that $\text{Best}(M, \mathcal{T}_O \cup T_2) = \epsilon$, that is, there exists a pair of states $s_1, s_2$ of $M$ with unobserved (for $\mathcal{T}_O \cup T_2$) paths $p_1, p_2$ from $s_1$ to $s_2$, such that the states traversed by $p_1$ and $p_2$ are disjoint, but for $s_1$ and $s_2$. We show now that $\text{Best}(M, \mathcal{T}_O \cup T_1) = \epsilon$.

If both $p_1$ and $p_2$ do not touch $C$, then $\text{Best}(M, \mathcal{T}_O \cup T_1) = \epsilon$. If both $p_1$ and $p_2$ belong to $C$, then $P_0(C, T_2)$ does
not hold, which means \( P_0(\mathcal{C}, T_1) \) does not hold, implying \( \text{Best}(M, \mathcal{T}_0 \cup \mathcal{T}_1) = \epsilon \).

If \( s_1, s_2 \in (Q' \setminus Q) \cup \{ s'_0, s'_f \} \), and \( \rho = \rho_1 \) or \( \rho_2 \) passes through \( C \), then there exists an unobserved (for \( \mathcal{T}_2 \)) path from \( s'_0 \) to \( s'_f \) (a subpath of \( \rho \)). Given this, \( P_1(\mathcal{C}, \mathcal{T}_2) \) does not hold; which implies that \( P_1(\mathcal{C}, \mathcal{T}_1) \) does not hold. Hence, there exists an unobserved (for \( \mathcal{T}_1 \)) path from \( s'_0 \) to \( s'_f \), and an unobservable (for \( \mathcal{T}_0 \cup \mathcal{T}_1 \)) path \( \rho' \) can be constructed from this path and \( \rho \). As such, there are two disjoint paths unobservable for \( \mathcal{T}_0 \cup \mathcal{T}_1 \) between \( s_1 \) and \( s_2 \): \( \text{Best}(M, \mathcal{T}_0 \cup \mathcal{T}_1) = \epsilon \).

If \( s_1, s_2 \in Q' \), and \( \rho = \rho_1 \) or \( \rho_2 \) passes through \( C \), then there exists an unobserved reverse cyclic pattern (for \( \mathcal{T}_2 \)) between \( s'_0 \) and \( s'_f \). Given this, \( P_1(\mathcal{C}, \mathcal{T}_2) \) does not hold; which implies that \( P_1(\mathcal{C}, \mathcal{T}_1) \) does not hold. Hence, there exists an unobserved (for \( \mathcal{T}_1 \)) reverse cyclic pattern between \( s'_0 \) and \( s'_f \), and an unobservable (for \( \mathcal{T}_0 \cup \mathcal{T}_1 \)) path \( \rho' \) can be constructed from this pattern and \( \rho \). As such, there are two disjoint paths unobservable for \( \mathcal{T}_0 \cup \mathcal{T}_1 \) between \( s_1 \) and \( s_2 \): \( \text{Best}(M, \mathcal{T}_0 \cup \mathcal{T}_1) = \epsilon \).

The cases where \( s_1 \in Q' \setminus \{ s'_0, s'_f \} \), \( s_2 \notin Q' \), or both \( \rho_1, \rho_2 \) pass through \( C \), are not possible because then the paths would meet in \( s'_0 \) and/or \( s'_f \).

Hence, \( \text{Best}(M, \mathcal{T}_0 \cup \mathcal{T}_2) = \epsilon \) \( \implies \) \( \text{Best}(M, \mathcal{T}_0 \cup \mathcal{T}_1) = \epsilon \). By symmetry between \( \mathcal{T}_1 \) and \( \mathcal{T}_2 \), we have the equivalence: \( \text{Best}(M, \mathcal{T}_0 \cup \mathcal{T}_2) \geq 0 \iff \text{Best}(M, \mathcal{T}_0 \cup \mathcal{T}_1) \geq 0 \). Now, for all \( x \in \{1, 1'\} \), we can enrich \( M \) to \( F_x(M) \) with \( M \) with \( \text{Best}(M, \mathcal{T}_0) \geq x \iff \text{Best}(F_x(M), \mathcal{T}_0) \geq 0 \). Applying it to \( M \), we get \( \text{Best}(M, \mathcal{T}_0 \cup \mathcal{T}_2) = \text{Best}(M, \mathcal{T}_0 \cup \mathcal{T}_1) = \epsilon \).

The functions \( F_x \) are given schematically in Figure 4, where \( F_x(M) = (Q', s'_0, s'_f, s'_f, \ldots) \).

For \( x \in \{0, 1, 1'\} \), we define \( \mathcal{T}_x(M) \) as a smallest subset \( \mathcal{T}_x \) of transitions of \( M \) such that \( P_x(M, \mathcal{T}_x) \) holds. For a subset of transitions \( T \) of a component \( C \) of \( M \), we also denote by \( \mathcal{T}_x^C(M) \) a smallest set \( \mathcal{T}_x \) such that \( \mathcal{T}_x \cap C = T \) and \( P_x(M, \mathcal{T}_x) \) holds. Every algorithm to compute the minimal observable set of transitions is recursive, taking the set of transitions considered observable as input. It is easy to modify them to input in the beginning not \( T \) but \( T' \), and disallowing to select any new transitions in \( C \), such that they compute \( \mathcal{T}_x^C(M) \), and they do it faster than \( \mathcal{T}_x(M) \) because they cannot choose among the transitions of \( C \). As proved in proposition 2, the size of \( \mathcal{T}_x \) is constant for several \( T \) such that \( \text{Best}(C, T) = y \).

If \( |T'| > |T| \) with \( \text{Best}(C, T) = \text{Best}(C, T') \), then \( |T_x^{T', C}(M)| > |T_x^{T, C}(M)| \). We can use this idea to compute \( \mathcal{T}_x(M) \) in a compositional manner, for a service \( M \) having component \( C \):

1. Compute a minimal set \( \mathcal{T}_0(C) \) of transitions of \( C \), \( \forall y \in \{0, 1, 1'\} \).
2. Compute a minimal set \( \mathcal{T}_1(C) \) of transitions of \( M \), \( \forall y \in \{0, 1, 1'\} \).
3. Output a set of smallest size among \( \mathcal{T}_0 \mathcal{T}_1(C) \).

For example, consider the service \( M \) having component \( C \) in Fig. 5.

1. A minimal set \( \mathcal{T}_0(M) = \{(s'_0, s_2), (s_1, s'_f)\} \), \( \mathcal{T}_1(M) = \{(s'_0, s_2), (s_1, s'_f)\} \), and \( \mathcal{T}_1(M) = \{(s'_0, s_2), (s_1, s'_f)\} \).
2. The corresponding observable sets of \( M \): \( \mathcal{T}_0^{T_0(C)}(M) = \{(s'_0, s_2), (s_1, s'_f), (s_0, s_f)\} \) of size 3, \( \mathcal{T}_1^{T_1(C)}(M) = \{(s'_0, s_2), (s_1, s'_f)\} \) of size 2, and \( \mathcal{T}_1^{T_1(C)}(M) = \{(s'_0, s_2), (s_1, s'_f)\} \) of size 3.
3. \( \mathcal{T}_0^{T_0(T_1(C))}(M) \) is a minimal observable set of \( M \).

We can now state the main theorem of the paper.

**Theorem 2** Let \( H = (M)_{i=1}^n \) be a hierarchical service. It is NP-complete in the size of \( H \) to compute \( MO(H) \). Moreover, it takes at most time \( O(\sum_{i=1}^n 2^{[M_i]}) \).

**Proof.** It suffices to compute a minimal set \( \mathcal{T}_e(M_v) \) of transitions of \( M_v \), for all \( v \in V = \{0, 1, 1'\} \). These actions are performed from bottom of the hierarchy to top. To compute \( \mathcal{T}_e(C_i) \) for a module \( M_i \) using \( M_j, M_k, j, k > i \), we use \( \mathcal{T}_e(M_j) \) and \( \mathcal{T}_e(M_k) \) which have already been computed. Indeed, it suffices to compute a minimal set \( \mathcal{T}_e^{T_0(C_i)}(M) \) of transitions of \( M \), for all
values $v', v''$. Then, it suffices to output a set among $T'_{M_1} \cup T'_{M_2} \cup (M_k \setminus M_k) \cup (M_k \setminus M_1 \setminus M_2) \cup (M_k \setminus M_1 \setminus M_2 \setminus M_3) \cup (M_k \setminus M_1 \setminus M_2 \setminus M_3 \setminus M_4) \cup (M_k \setminus M_1 \setminus M_2 \setminus M_3 \setminus M_4 \setminus M_5) \cup (M_k \setminus M_1 \setminus M_2 \setminus M_3 \setminus M_4 \setminus M_5 \setminus M_6) \cup (M_k \setminus M_1 \setminus M_2 \setminus M_3 \setminus M_4 \setminus M_5 \setminus M_6 \setminus M_7) \cup (M_k \setminus M_1 \setminus M_2 \setminus M_3 \setminus M_4 \setminus M_5 \setminus M_6 \setminus M_7 \setminus M_8) \cup (M_k \setminus M_1 \setminus M_2 \setminus M_3 \setminus M_4 \setminus M_5 \setminus M_6 \setminus M_7 \setminus M_8 \setminus M_9)$ of minimal size.

It is important to notice that since a service is in particular a hierarchical service (with hierarchy height of 1), we know that the problem is at least NP-hard. However, the complexity could be exponentially worse for hierarchical graphs, since a small hierarchical graph can represent an exponentially bigger flat graph. We prove that this is not the case. Moreover, we prove that the complexity is linear in the number of components, and exponential only in the size of each base component. That is, we prove that with a smart algorithm, one can compute efficiently the absolute minimal observable size even for huge hierarchical systems, as long as each component is small enough. The best case comparison is with respect to a hierarchical service of diameter $O(2^n)$, having many base components of size 2 (each one being reused $2^{n-1}$ times). The brute force non-compositional method runs on $H$ and takes time $O(2^{2^n})$, while our method takes $O(n)$, that is a doubly exponential improvement (one exponential due to the reuse of components, and another due to decomposition).

### 6 Folding Unfolded Structures

The problem is that the complex services to analyze is not often given in a hierarchical way. Moreover, there is no clear hierarchical description of a service. Instead, we explain here how to find or recover a hierarchical structure from a flat service.

We present here a linear time (in the number of transitions) algorithm to compute the smallest components $C$ of an FSM $M$ knowing its starting and ending state, and an initial transition from starting state.

**Input.** A service $M = (Q, s_0, s_f, T)$, $\tau = (s, s_1) \in T$ and $t \in Q$.

**Output.** The smallest component $C = (Q', s, t, T')$ of $M$ with $\tau \in T'$.

**Initialization.** $T' = \emptyset$, $S = \{\tau\}$, $Q' = \{s, s_1, t\}$.

1. Select a transition $\tau' = (s_1', s_2') \in S$. If $s_2' \neq t$, then $S = S \setminus \{\tau'\} \cup \{s_1', s_2'\} \in T \setminus T'$. If $s_1' \neq s$, then $S = S \cup \{(s_1', s_1')\} \cup \{s_2', s_1'\} \in T \setminus T'$. Finally, $S = S \setminus \{\tau'\}$, $T' = T' \cup \{\tau'\}$, and $Q' = Q' \cup \{s_1', s_2\}$.

2. If $S \neq \emptyset$, repeat step 1.

3. If $(s \neq s_0 \land s_0 \in Q') \lor (t \neq s_f \land s_f \in Q')$, then return that a component between $s$ and $t$ with respect to $\tau$ does not exist. Else, return $C$.

The above algorithm can be iteratively invoked to compute the set $S_C$ of all components of an FSM $M$. We now give an algorithm to compute the largest component of $M$.

1. For a pair of components $D, E \in S_C$, if $D$ is a subgraph of $E$, delete $D$.

2. For a pair of components $D = (Q', s_0', s_f', T')$ and $E = (Q'', s_0'', s_f'', T'')$ of $S_C$, if $s_0' = s_0''$ and $s_f' = s_f''$, then create a new component $F = (Q' \cup Q'', s_0', s_f', T' \cup T'')$. If $F \neq M$, then delete $D$ and $E$ from $S_C$, and add $F$ to $S_C$.

3. Return the biggest $C \in S_C$.

**Proposition 3** The biggest component of a system $M$ is computed in quadratic time using the above algorithm.

The procedure can thus be inductively called until there is no more component in the graph, and then the hierarchical structure of the graph is obtained.

### 7 Experimental Results

We tested our decomposition algorithm on hierarchical graphs. First, we choose a number (between one and nine) of base subcomponents in the graph. Then, we generate each of them randomly by using the Synthetic DAG generation tool (http://www.loria.fr/~suter/dags.html). We then generate inductively a hierarchical graph having these base subcomponents randomly using the same tool, by assigning two edges to these components. There is no reuse of components. For each value, we generate each hierarchical graph and each base subcomponent five times to compute the mean values (because of variation in runtime and observable size). We then unfold the hierarchical graphs as (flat) graphs, whose size is linear in the number of base components. We then run both a brute force algorithm and our hierarchical algorithm on these graphs. We do not input the hierarchical shape of the graph, instead the algorithm finds the optimal decomposition. With the polynomial time folding algorithm. Fig. 6(left) shows the times (in logarithmic scale) needed to compute a minimal observable set using brute force and our decomposition algorithm wrt. the number of edges (which is linear wrt. the number of base subcomponents).

![Figure 6: Execution time & observable size](image-url)
Our decomposition algorithm is indeed linear time wrt. the number of base subcomponents/number of edges (0.14s for an average number of edges of 18 and 0.73s for an average number of edges of 108), while the brute force is exponential in the number of edges, already timing out at a little over 40 edges. For 1 subcomponent, the overhead of our method makes the decomposition slightly worse than the brute force method. Fig. 6(right) shows the percentage of edges needed to be logged among all the edges. Both algorithms answer the same number on the same graphs but there is a huge variation among graphs, from one edge needs to be logged out of 4 to one edge out of 15. The mean value seems to tend to one out of 6.

8 Distributed Services

We consider distributed services, given in the form of product of two services $M$ and $N$. We first on services having no interaction between them (that is, one service cannot write a global variable that is read or written by another service). We explain later how to deal with interacting services having a non interacting component.

The composition schema of such a composite service is specified as a product of the FSM’s (corresponding to the composition schema) of the component composite services $[3, 6]$. Given services $M = (Q_1, s_0, s_f_1, T_1)$ and $N = (Q_2, s_0, s_f_2, T_2)$, $T_1 \cap T_2 = \emptyset$, we define their product $M \times N = (Q, s_0, s_0, s_f, s_f, T)$ with $Q = Q_1 \times Q_2$ and

$$ T = T_1 \times T_2 \cup T_2 \times T_1 $$

where $T_{hs}$ is the set of self loop transitions $(s_j, s_i)$ for $s_i \in Q_k$, $k = 1, 2$.

The observation and logging for each component service are done locally. Hence, our decomposition method cannot be applied on the product since choosing to observe a transition in a component of the product might force it to be observed in another as well (if the same component is reused). Moreover, strict execution sequence detection is not required, since not knowing the exact interleaving between two “equivalent paths” is not needed. More formally, two consecutive transitions $\tau_i$ and $\tau_{i+1}$ of a path $\rho$ of $M \times N$ can commute if $\tau_i$ is from $M$ and $\tau_{i+1}$ from $N$, or vice versa. Two paths $\rho_1 \neq \rho_2$ of $M \times N$ are equivalent $\rho_1 \equiv \rho_2$, if $\rho_1 = \rho_2$ after a finite sequence of commutations on $\rho_1$ (or $\rho_2$). We say that the product $M \times N$ is observation sequence detectable with the observation of $T_i$ in $M$ and $T_2$ in $N$ if for all paths $\rho_1, \rho_2$ of $M \times N$ such that $\text{Obs}_{T_i \cup T_2}^{\text{last}}(\rho_1) = \text{Obs}_{T_i \cup T_2}^{\text{last}}(\rho_2)$, we have $\rho_1 \equiv \rho_2$. In this case, compensation can be performed using any of the equivalent (reversed) runs, this will result in the same consistent state. We then have the following desirable property:

**Proposition 4** For a pair of non interacting FSM’s $M$ and $N$, and one of their respective minimal observable sets $T_{OM}$ and $T_{ON}$. $T_{OM} \cup T_{ON}$ is a minimal observable set of $M \times N$.

Now, let us consider the more general case where $M$ and $N$ interact, but $M$ has a component $C$ that does not interact with $N$. Then, one can decompose $M$ into $C$ and $M \setminus C$, and compute an observable set $T_i$ of transitions of $C$. We can choose the observations of $C \times T_{2e}$ to be one of the $T_i$.

**Example 2** We consider FSM’s $M = (Q_1, s_0, s_f, T_1)$ (Fig. 7a) and $N = (Q_2, s_0, s_f, T_2)$ (Fig. 7b) representing e-services which allow searching and listening to songs online $[3]$. The e-services allow different modes of payment and searching for song files by singer/title. Their product $M \times N$ is shown in Fig. 7c. The FSM $M$ contains a component $C = (Q', s_0, s_1, T')$ such that $T' \cap T_2 = \emptyset$. Now, let us consider minimal observable sets $T_{O_1} = \{a\}$ and $T_{O_2} = \{e\}$ of $C$ and $M \setminus C \times N$, respectively. Then, $T_{O_1} \cup T_{O_2}$ is a minimal observable set of $M \times N$ as shown in Fig. 7c (with the dashed arrows representing observable transitions).

9 Discussion and Conclusion

We studied compensation under partial log visibility. To the best of our knowledge, this problem has never been considered in the context of transactional services. We proposed a framework which uses the hierarchical nature of
composite services, and gives an efficient algorithm to compute the absolute minimum number of transitions to observe in order to get compensability.

The algorithm we proposed considers only a subset of the whole set of transitions. It is thus straightforward to add constraints, such as, a subset of transitions “can/cannot be observed”. It is very useful since in practice, we have to take into account privacy/security issues. The algorithm would then answer the absolute minimal observable set among those satisfying the constraints. Also, the hierarchical decomposition allows to deal with dynamicity. Indeed, if a service gets transformed (e.g., after the discovery/death of a sub-service), obtaining a minimal observable set would need recomputation, only at its level of the hierarchy (not below), plus few levels above (until the properties of a level are unchanged). It also allows to describe more accurately the details of a service which was considered atomic until now, in order to have feedback on where a service failed exactly. We can also deal with distributed services, and with systems which are not given in a hierarchical way (using our folding algorithm).

References