Application of the Causal Ordering Graph to Synchronous Motors, Part I: Model Description

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Abstract: - Nowadays, the control design of synchronous motors is well-known. So this facilitates the presentation of graphical tools of model description and control design such as the Causal Ordering Graph (COG). This paper is Part I of a two-part article. In Part I, we present classical models of permanent magnet synchronous motors using the COG. These models are developed and analysed in the stationary reference frame, in Concordia’s reference frame and in Park’s reference frame. The Causal Ordering Graph helps to determine the nature of the interactions between currents and fluxes. Moreover, the torque expression of synchronous motors is presented as a non-bijective relation. In order to establish nonspecific models, an analogy between rotary and linear synchronous motors is presented. Afterwards, the specificities of linear motors are exposed. In Part II, the models represented using the COG will be used to generate control structures using the inversion principle of the COG.

Key-Words: - Causal Ordering Graph, Rotary Synchronous Motor, Linear Synchronous Motor, Modelling.

1 Introduction
The purpose of this paper is to present a systematic method for system modelling and control design. Generic models of synchronous motors are established using the Causal Ordering Graph (COG) in order to design the optimal control structures of such systems.

Today, numerous modelling techniques are available: finite elements models, self-tuning parameter models, state space formalism, etc [1]. Nevertheless, some of these models can’t be used directly inside control structures, because of: implementation limitations, maximum value of CPU turnaround time, etc [2]. In this paper, we apply the COG formalism to a synchronous motor because it takes into account the causal behaviour of physical phenomena. In fact, the COG is a descriptive method, which helps to discern the causality of a system [3]. This property becomes truly important to elaborate control strategies for maintaining the torque control.

After explaining the principle of the Causal Ordering Graph, we apply this methodology to simplified models of synchronous motors in (α-β) Concordia’s reference frame and in (d-q) Park’s reference frame. The (abc) stationary reference frame gives a so-called natural model, whereas (α-β) Concordia’s transformation gives a so-called generalised model. (d-q) Park’s reference frame, known as the synchronous reference frame, gives constant values of corresponding currents and fluxes in the direct axis and quadrature axis only.

Then, the nature of linkage between currents and fluxes is explained. Moreover, the torque expression is explained as a non-bijective relation that confers some degree of freedom for the control design in this relation.

Furthermore, as we develop a generic model, an analogy between rotary and linear synchronous motors is made. The specificities of linear motors like the longitudinal end-effects, the mutation of the harmonics value of the back electromotive force and the unbalanced inductances are exposed and detailed in order to obtain more accurate models.

In Part II, the inversion principle of the COG is applied so as to build an optimal control structure. Thus, examples of vector control strategies in Concordia’s reference frame and in Park’s reference frame are given, and validated with experimental results on a linear synchronous motor.

2 The Causal Ordering Graph: Model
2.1 The COG specificities
The design of optimal controllers is a classical engineering issue. A system modelling help engineers to understand how a system works and consequently facilitate the design of adapted control strategies. But, to study a complex system, only systematic methods guarantee the success of the analysis. The Causal Ordering Graph is a methodical approach based on a located energy representation with the theory of causal ordering [3-6].

It is undeniable that this approach is connected with that of links graphs such as the Bond Graphs [7-11]. But it differs from them by the analysis process, which is based on integral causality only. The COG is a tool that structures the synthesis of a state model. But, for a given
system, that aims at maintaining a representation as close as possible to that felt in the observation. Obviously, the resulting model will be affected by the neglected elements as well as the explicit and implicit assumptions induced by the physical interpretation of the constitutive objects. Lastly, the fundamental objective of the COG tool is to propose, by a graphical method and thanks to some simple principles, a synthesis of the control law taking into account the physical transfers of the process.

To summarize, the interest of the COG is to define ordering models and its originality is to provide the inverse model which focuses the designer on the required operations to control the process.

2.2 The Causal Ordering Graph principle

The Causal Ordering Graph is built up with several graphical processors attached to different objects located in the studied process. The evolution of these objects is characterized by a transformation relation between influencing quantities and influenced quantities. This relation is induced by the principle of causality governing the energetic relation of an object or group of objects. In short, the output of a processor only depends on present or past values of the inputs. Such a formulation expresses the causality in integral form, and many significant electrical and mechanical examples illustrate this concept. Since the flux in a self is an integral function of the voltage, by analogy, the kinetic moment of a rigid mass is the integral function of the applied efforts. The electricity quantity in a capacitor is an integral function of the current; by analogy, the endpoints position of a spring is the integral of the velocity variation between the endpoints (Hooke’s law) [12].

In general, the expression of the transformation relations by means of the state equations is the best warranty against physical misinterpretation. To simplify the presentation, we will retain only two complementary definitions of integral causality (Fig.1.a). If an object accumulates information, causality is internal: the relation, thus oriented, is known as causal. Time and the initial state are implicit inputs and are not represented (Fig.1.b). If an object does not accumulate information, causality is external. The output is an instantaneous function of the input. The relation, which is not oriented, is then known as rigid. Fig.1 gives the selected symbolism to differentiate between the two kinds of processors.

\[ R \rightarrow y(t) = (y(t))_o + \alpha(u) \int_0^t u(t) dt \]  
(a) \[ R \rightarrow y(t) = \alpha(u) t \]  
(b)

Fig.1: COG symbolisms: (a) causal relation, (b) rigid relation.

3 Model of Synchronous Motors

3.1 Assumptions and definitions

To focus the work presented here, the following assumptions are made:

1) Only three-phase motors are considered.
2) Motors are star-connected with an inaccessible neutral wire.
3) All three phases are balanced, so currents have an electrical angle offset value of \(2\pi/3\) with respect to one another.
4) The same goes for the back electromotive forces.
5) Resistances and inductances of the three phases are identical and constant.
6) The magnetic circuit is not saturated.
7) The magnets are surface-mounted on the rotor, with a constant air-gap.
8) Slot effects are taken into account as cogging force with its first harmonic value.
9) Magnetomotive forces are considered to have a sinusoidal space repartition.

Table 1: List of symbols and Parameters name

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter name</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_{abc})</td>
<td>Parameter X in the abc reference frame</td>
</tr>
<tr>
<td>(V_X)</td>
<td>Electric voltages in the X reference frame</td>
</tr>
<tr>
<td>(i_X)</td>
<td>Currents in the X reference frame</td>
</tr>
<tr>
<td>(R)</td>
<td>Armature winding resistances per phase</td>
</tr>
<tr>
<td>(L)</td>
<td>Self Inductance per phase</td>
</tr>
<tr>
<td>(M)</td>
<td>Mutual inductance</td>
</tr>
<tr>
<td>(\dot{\phi}_f)</td>
<td>Maximum value of magnetic excitation flux</td>
</tr>
<tr>
<td>(J)</td>
<td>Rotor inertia</td>
</tr>
<tr>
<td>(T_{em})</td>
<td>Electromagnetic Torque</td>
</tr>
<tr>
<td>(T_f, T_{cog})</td>
<td>Friction force and Cogging force</td>
</tr>
<tr>
<td>(T_r)</td>
<td>Resistance force</td>
</tr>
<tr>
<td>(\tau_p)</td>
<td>Pole pitch: Step between two magnetic poles</td>
</tr>
<tr>
<td>(N_p)</td>
<td>Electrical position constant</td>
</tr>
<tr>
<td>(\theta_e)</td>
<td>Electrical angle</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Mechanical angle of the rotor</td>
</tr>
<tr>
<td>(\omega)</td>
<td>Electrical angular speed</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>Mechanical angular speed of the rotor</td>
</tr>
<tr>
<td>(x)</td>
<td>Linear displacement</td>
</tr>
<tr>
<td>(v)</td>
<td>Linear velocity</td>
</tr>
</tbody>
</table>
Depending on construction, materials and rotor design, we can classify synchronous motors in four basic groups: reluctance motors, electromagnetically-excited motors, hysteresis motors, and permanent magnet (PM) motors [13-14]. At this point, the study is restricted to PM motors only. The polyphase synchronous motor considered has three pole pairs. The rotor is made of surface-deposited permanent magnets. Fig.2 gives a scheme of the permanent magnet synchronous motor (PMSM) studied.

![Schematic view of the studied PMSM](image)

Fig.2: Schematic view of the studied PMSM

Consequently, we can apply additive properties on fluxes and currents. Then, axes are defined in the electrical frame, where \( \theta = N_p \cdot \theta \).

Three identical windings in the stator are separated from each other with an electrical angle of \( 2\pi / 3 \). The axis of phase \( a \) defines the \( (abc) \) stationary reference frame, Fig.2.

Moreover, the \( a \) axis also represents the direct axis \( a \) of Concordia’s reference frame. The quadrature axis \( \beta \) of Concordia’s reference frame is separated by a mechanical angle of \( \pi / 6 \) from its direct axis \( a \). Concordia’s reference frame is a so-called diphase stationary reference frame.

Inside the rotor, the direct axis \( d \) of Park’s reference frame is defined in the middle of a north permanent magnet. Then, the quadrature axis \( q \) of Park’s reference frame is separated by a mechanical angle of \( \pi / 6 \) from its direct axis \( d \) (the same as an electrical angle of \( \pi / 2 \)). Park’s reference frame is a so-called synchronous reference frame.

### 3.2 The abc stationary reference frame

Equations of electrical part of surface-mounted PM synchronous motors are depicted as follows:

\[
[R_{abc}] = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix}, \quad [L_{abc}] = \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix}
\]

(1)

\[
\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = [R_{abc}] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + [L_{abc}] \begin{bmatrix} \frac{d}{dt}i_a \\ \frac{d}{dt}i_b \\ \frac{d}{dt}i_c \end{bmatrix} + \omega N_p \begin{bmatrix} d\phi_e \\ d\phi_e \\ d\phi_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

(2)

\[
T_{em} = e_a \cdot i_a + e_b \cdot i_b + e_c \cdot i_c
\]

(3)

Fig.3 presents the Causal Ordering Graph of the electrical part in the stationary reference frame. The COG relations of Fig.3 show that balanced conditions give a symmetrical graph, and explain that phases can be represented as a simplified vector scheme.

![Causal Ordering Graph of the electrical part of a PMSM in (abc)](image)

Fig.3: COG of the electrical part of a PMSM in (abc)

The different relations are given by:

R2 \( \rightarrow V_{labc} = V_{abc} - e_{abc} = V_{abc} \)

R3 \( \rightarrow (L - M) \frac{d}{dt}i_{abc} = V_{labc} \)

R5 \( \rightarrow T_{em} = i_a \cdot \frac{d\phi_e}{d\theta} + i_b \cdot \frac{d\phi_e}{d\theta} + i_c \cdot \frac{d\phi_e}{d\theta} \)

R6 \( \rightarrow \frac{d\phi_e}{d\theta} = N_p \phi_e \sin(N_p \cdot \theta - \pi / 3) \)

R7 \( \rightarrow e_{abc} = \omega \frac{d\phi_e}{d\theta} \)

With the assumptions taken about the inductances and mutual inductances, the inductance voltage \( V_I \) of the R3 processor corresponds to an equivalent voltage for the \( L-M \) inductance, which represents a part of the energy stored in the system.

The R1 processor represents a matrix transformation used to respect the effect of a non-distributed neutral wire, especially to eliminate homopolar components and currents with harmonic ranks that are multiples of three. We can notice that this matrix transformation is not necessary inside the R6 processor because the flux relations are already sinusoidal without harmonic ranks that are multiples of three.

\[
\begin{cases}
V_a = \frac{1}{3}(2V_{SN} - V_{SN} - V_{SN}) \\
R1 \rightarrow V_b = \frac{1}{3}(2V_{SN} + 2V_{SN} - V_{SN}) \\
V_c = \frac{1}{3}(2V_{SN} - V_{SN} - V_{SN})
\end{cases}
\]

The R1 processor explains that injection of third rank harmonics of voltages has no impact on the torque, but could give unsaturated fluxes of the motor.
Next, the mechanical model of the motor can be represented as in Fig.4:

![Fig.4: COG of the mechanical part of a PMSM](image)

The different relations are given by:

\[
\begin{align*}
R8 & \rightarrow T_{em} = T_m - T_f - T_{cog} \\
R9 & \rightarrow J \frac{d}{dt}\dot{\omega} = T_{motor}, \quad R11 \rightarrow \left( \frac{d}{dt} \right)\dot{\theta} = \omega \\
R12 & \rightarrow T_{cog} = \dot{T}_{cog} \cdot \sin(6N_p \cdot \theta)
\end{align*}
\]

The \(R10\) processor represents friction phenomena, which are non-linear. Depending on the control strategy, if speed control is needed at a high-speed value, viscous friction and Coulomb’s friction should be taken into account. Otherwise, if speed control is needed at a low speed value, Stribeck’s phenomenon, for example, could be included, [15].

The \(R12\) processor represents the cogging forces induced by the interaction between the PMs mounted on the rotor and the stator anisotropy, due to the slotting. According to the stator design of Fig.2, with one magnet for three slots and three teeth, the fundamental harmonic of the cogging forces frequency is six times superior to the current frequency [16]. Yet, motors with closed slots or slotless stators are not affected by cogging torque.

As noticed in the COG relations of Fig.4, all the relations are independent of electrical reference frames. So, the mechanical model of a PMSM will be exactly the same in the next developments.

3.3 The \(\alpha\-\beta\) diphase stationary reference frame

Most of contemporary industrial applications with synchronous motors are made of three-phase windings, which are star-connected with an inaccessible neutral wire. So, the homopolar component is eliminated. Furthermore, the three currents are linked. Consequently, only two supplied voltages are truly needed to control the PMSM. So, we apply \((\alpha\-\beta)\) Concordia’s reference frame to (1-3) [17]. Concordia’s transformation is defined by:

\[
\begin{align*}
[f_{\alpha\alpha}] &= [C_3^*] \cdot [f_{\alpha\beta}] \quad \text{with} \quad C_3^* = \frac{2}{\sqrt{3}} \begin{bmatrix}
1 & 0 & 1/\sqrt{2} \\
-1/2 & \sqrt{3}/2 & 1/\sqrt{2} \\
-1/2 & -\sqrt{3}/2 & 1/\sqrt{2}
\end{bmatrix} \\
[f_{\alpha\beta}] &= [C_3] \cdot [f_{\alpha\alpha}], \quad \text{with} \quad C_3 = \frac{2}{\sqrt{3}} \begin{bmatrix}
1 & -1/2 & -1/2 \\
0 & \sqrt{3}/2 & -\sqrt{3}/2 \\
1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2}
\end{bmatrix}
\end{align*}
\]

Finally, by transforming (6) and (9):

\[
\begin{align*}
T_{cog} &= \frac{3}{2} N_p \hat{\phi}_r \left[ \cos(\theta_r) \cdot i_\beta - \sin(\theta_r) \cdot i_\alpha \right] \\
T_m &= \sqrt{3}/2 \cdot N_p \hat{\phi}_r \left[ \cos(\omega t + \Psi) - \cos(2\omega t + \Psi) \right] \\
T_p &= \sqrt{3}/2 \cdot N_p \hat{\phi}_r \left[ \cos(\Psi) + \cos(2\omega t + \Psi) \right]
\end{align*}
\]
It appears that each elementary machine gives half of the torque value, plus a pulsating component with the same amplitude value, but with a frequency which is twice that of the speed rotating frequency. These classical results show how the implicit mechanical linkage between the two fictive machines eliminates the pulsating component.

### 3.4 The d-q synchronous reference frame

Park’s transformation is defined by:

\[
[f_{oc}] = [T_1] [f_{aq}], \text{ and } [f_{aq}] = [T_1^{-1}] [f_{oc}]
\]

with \( T_1 = \sqrt{3} \begin{bmatrix}
\cos(\theta_i) & -\sin(\theta_i) & 1/\sqrt{2} \\
\cos(\theta_i - 2\pi/3) & -\sin(\theta_i - 2\pi/3) & 1/\sqrt{2} \\
\cos(\theta_i - 4\pi/3) & -\sin(\theta_i - 4\pi/3) & 1/\sqrt{2}
\end{bmatrix},
\]

and \( T_1^{-1} = \frac{2}{\sqrt{3}} \begin{bmatrix}
\cos(\theta_i) & \cos(\theta_i - 2\pi/3) & \cos(\theta_i - 4\pi/3) \\
-\sin(\theta_i) & -\sin(\theta_i - 2\pi/3) & -\sin(\theta_i - 4\pi/3) \\
1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2}
\end{bmatrix} \).

Fig.6 presents the Causal Ordering Graph of the electrical part in the synchronous reference frame:

![Causal Ordering Graph](image)

The different relations for the flux axis are given by:

\[
R2d \rightarrow V_{id} = V_q - e_q - V_{id}, \ R7d \rightarrow e_q = k_q \cdot \omega
\]

\[
R3d \rightarrow L_d \cdot \frac{di}{dt} = V_{id}, \ R4d \rightarrow V_{id} = R \cdot i_d
\]

\[
R5d \rightarrow T_d = k_d \cdot i_d, \ \ R6d \rightarrow k_d = -N_p \cdot (L_d \cdot i_d)
\]

The different relations for the torque axis are given by:

\[
R2q \rightarrow V_{iq} = V_q - e_q - V_{iq}, \ R7q \rightarrow e_q = k_q \cdot \omega
\]

\[
R3q \rightarrow L_q \cdot \frac{di}{dt} = V_{iq}, \ R4q \rightarrow V_{iq} = R \cdot i_q
\]

\[
R5q \rightarrow T_q = k_q \cdot i_q, \ \ R6q \rightarrow k_q = N_p \cdot (L_q \cdot i_q + \sqrt{3}/2 \cdot \dot{\phi})
\]

The R1 processor represents Park’s transformation matrix described in (11). The benefits of Park’s transformation are the conversion of the sinusoidal values of voltage, fluxes and currents into constant values, which make the controller design easier. That’s represented in Fig.6 with the presence of the electrical angle in the R1 processor. So, this sinusoidal envelop is reported outside the electrical model in the R1 processor as a simple transformation operator.

The R8 processor includes the torque expression, which is defined from two elementary machines (13). We can notice that the direct axis \( d \) represents the flux axis, and the quadrature axis \( q \) represents the torque axis. Thus, we can say that each machine looks like a DC motor.

\[
T_{em} = T_d + T_q = \phi_k \cdot i_d - \phi_k \cdot i_d
\]

Finally, as the motor is balanced and inductances from these flux loops will be more complicated, especially in case of saturation [18].

\[
T_{em} = T_d + T_q = N_p \cdot (L_d - L_q) \cdot i_d + N_p \cdot \sqrt{3}/2 \cdot \dot{\phi} \cdot i_q
\]

Finally, as the motor is balanced and inductances from these flux loops will be more complicated, especially in case of saturation [18].

\[
T_{em} = T_d + T_q = N_p \cdot \sqrt{3}/2 \cdot \dot{\phi} \cdot i_q
\]

The R5d and R7d processors denote a gyrator. The gyrator in COG formalism is used as a symbol of all electromechanical conversions. The variables of dual energy nature are modulated by a coefficient \( k \) that is characteristic of the object; the gyrators associate processes of comparable energy nature. Here, the coefficients \( k_d \) and \( k_q \) are defined by the R6d and R6q processors respectively. The disadvantage of Park’s transformation is that the coefficients \( k_d \) and \( k_q \) needed for the gyrator denote a cross coupling between the two axes. Consequently, in the controller design, the control of these flux loops will be more complicated, especially in case of saturation [18].

### 3.5 Conclusion

We have obtained electrical models in the stationary reference frame (\( abc \)) and in Concordia’s reference frame (\( \alpha\beta \)). The Causal Ordering Graph of each model has exactly the same structure. Obviously, processor relations are different, depending on the applied transformation matrix. So, in the stationary reference frame, the three currents are sinusoidal with an offset of \( 2\pi / 3 \) on each other. Thus, for a closed-loop control strategy, controllers have to compensate errors on sinusoidal signals.

In Concordia’s reference frame, there are only two sinusoidal currents in quadrature. So, two controllers only will be required to design the torque control loop.

Finally, we have obtained an electrical model in Park’s reference frame. The two currents \( i_d \) and \( i_q \) are constant at a constant speed. But only one current \( i_q \) is really needed to assume the torque control. Indeed, if the second current \( i_d \) is set to zero, then the torque turns into \( T_{em} = K \cdot i_q \), which will clearly reduce the control structure.
4 Rotary / linear motor analogy

Nowadays, in motion control of linear system, the performances of mechanical transmission systems classically limited by ball lead screws and gearing are improved with the use of linear motors due to advances in electronics and PM quality [19]. Permanent magnet linear synchronous motors (PMLSM) are now widely used in machine tools applications [20]

4.1 Structure of a linear synchronous motor

Fig.7 represents an example of PM linear synchronous motor geometry: A LIMES400/120 by Siemens. This PMLSM is used in an application of XY high speed laser cutting. The performances of the system are about 300m/min and 40m/s² with a moving mass of 350Kg.

Fig.7: Picture of the LIMES400/120 (Siemens)

Fig.8 represents the principle schematic of the LIMES400/120 by Siemens. The moving part is called the forcer. It’s composed of a three-phase winding called the primary. The unmoving part is called the field excitation system. It’s composed of an array of magnetic poles N, S… N, S.

$\tau_p$ represents the pole pitch step between two consecutive magnetic poles of the secondary [18].

Fig.8: Principle schematic of a PMLSM (Siemens)

The principle schematic of the LIMES400/120 is simplified, as we have made some assumptions:
1) The pole shoes are not represented;
2) The design of the primary extremities is not shown;
3) The disposition of the magnets is not skewed.

4.2 Principle of the rotary / linear analogy

For a rotary motor, the three armature windings are each shifted by an electrical angle of $2\pi / 3$, and each winding covers an electrical angle of $\pi$ in the stationary reference. By analogy, the stator armature windings of a PMLSM are each shifted by a distance of $2 \tau_p / 3$, and every winding covers a distance of $\tau_p$ in the linear reference frame.

The electrical angle along which the primary of the PMLSM moves in the linear reference frame can be expressed by:

$$\theta_e = N_p \cdot x, \quad \text{with} \quad N_p = \frac{\pi}{\tau_p} \quad (16)$$

Here, $N_p$ is the electrical position constant of the PMLSM. Equivalences of the electrical angle and the electrical angular speed between rotary motors and PMLSMs are shown in Table 2.

Table 2: Equivalences of electrical angle and electrical angular speed between rotary motors and PMLSMs

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Rotary motor</th>
<th>PMLSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical angle ($\theta_e$)</td>
<td>$\theta_e = N_p \cdot \theta$</td>
<td>$\theta_e = N_p \cdot x$</td>
</tr>
<tr>
<td>Electrical angular speed ($\omega$)</td>
<td>$\omega = N_p \cdot \Omega$</td>
<td>$\omega = N_p \cdot \nu$</td>
</tr>
</tbody>
</table>

Classically, in machine tool applications, the linear motors are star-connected with an inaccessible neutral wire: only two currents are independent [21]. Thus, the ($\alpha$-$\beta$) reference frame is more appropriate to represent such systems.

4.3 More detailed model

In this section, we can go further into our PMLSM model. Indeed, among the assumptions we have taken, we have considered that the geometry of a linear motor is the same as that of a rotary motor, such as the ratio of the number of slots by the magnets width.

Nevertheless, models of linear motors can be different from models of rotary motors. Thus, for example, we can find:

1) Longitudinal end-effects forces are added to cogging forces, which turn into detent forces [22]

$$R12 \rightarrow T_{det} = \hat{T}_{cog} \cdot \sin(6N_p \cdot \theta) + \hat{T}_{det} \cdot \sin(2N_p \cdot \theta) \quad (17)$$

The longitudinal end-effects have a period which is directly connected with the periodicity of the magnet. We can notice in Fig.7 that for the LIMES400/120 the longitudinal end-effects are reduced by a special design of extremities of the primary.
2) Back-electromotive forces have harmonics mutation, as in the R6 processor of Fig.5 [23]:

\[
R6 \rightarrow \frac{d\phi_{el}}{dx} = \frac{3}{2} N_p \phi_0 \sum_{n=-\infty}^{\infty} K_{\alpha}^{n+1} \sin \left[ \frac{(2n-1)N_p}{2} \cdot x \right] \]

The harmonics of back-electromotive forces are induced by different causes:
- The slots design of the primary: with or without pole shoes;
- The magnet disposition on the yoke: the yoke plate is or isn’t fragmented, which changes the magnet flux path.

For the LIMES400/120, the harmonics of Back-EMF are about: 20% of third rank harmonics (H3/H1), and 3% of fifth rank harmonics (H5/H1).

3) Balanced inductances (1) are changed to unbalanced inductances [24]:

The longitudinal end effects introduce an asymmetry on the magnetic field. Consequently, the mutual inductances are different, which yields different values for the cyclic inductances. This effect is more important in the motors with few pole pairs. Some manufacturers give improvements to the windings and then avoid the asymmetry of the machine, but these techniques are limited. Hence, to point out the asymmetric phenomenon in the linear machines, some studies have been carried out using a numerical model based on the 3D finite element method [25].

\[
[L_{\alpha\beta}^T] = \begin{bmatrix}
L & M_1 & M_1 \\
M_1 & L & M_2 \\
M_2 & M_2 & L
\end{bmatrix}
\]

The asymmetry could be about 20% between the different mutual inductance [24].

5 Conclusion
In this paper, we have presented the modelling of a synchronous motor in order to design an optimal control structure. This Part I presents the electrical model of a PMSM in the (abc) reference frame, the (a-b) stationary reference frame and the (d-q) reference frame. Each time, the Causal Ordering Graph (COG) formalism has been applied to analyse the waveforms and the interactions between the voltages, the currents and the fluxes. We have shown that our models can be used with rotary and linear synchronous motors. Some of the specificities of linear motors have been explained, such as the longitudinal ends-effects, the detent force and the unbalanced inductance.

The Causal Ordering Graph is a powerful tool to describe and analyse a system, like the Bond Graph formalism. Complex systems including mechanical elements, electrical elements, etc., could be modelled using the COG, and finally depicted as a comprehensive graph [26]. Then, the main originality of the Causal Ordering Graph tool is the facility of designing the optimal control structure for a system. In Part II, we present the control design by applying the inversion principle of the COG to our PMLSM models.

References:


