Finite-Element Analysis of Non-Sinusoidal Electromotive Force in a Permanent Magnet Linear Synchronous Motor

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Abstract In the case of permanent magnet linear synchronous motors (PMLSM), knowledge of the electromotive force (emf) is necessary to achieve accurate control. However, it is often difficult to obtain it experimentally. As a consequence, this paper deals with the study of the non-sinusoidal emf of a PMLSM using the Finite-Element Method (FEM). Two potential formulations are used to consider several cases. Then, for the same accuracy, the computation times are compared. The results of the FEM are compared to those given by the measurements from a test bench composed of a Siemens LIMES400/120 linear motor and of a dSPACE DS1005 real-time controller board. Finally, the effectiveness and the limits of the FE Analysis will be discussed.

Keywords: PMLSM, Non-Sinusoidal Electromotive Force, Finite-Element Analysis

1. Introduction

To control permanent magnet linear synchronous motors (PMLSM) in an optimal way, the knowledge of the real waveform of different electrical characteristics is required. Thus, control strategies can be built on accurate values and yield reliable processes, notably on the thrust control. One of the electrical characteristics needed is the electromotive force (emf). This can be sinusoidal or not, which implies different control approaches. Furthermore, it is generally difficult to obtain this emf from measurements. Nowadays, the three-dimensional finite-element method (3D-FEM) is widely used to model PMLSMs (1)~(4). This approach takes into account the real geometry of the modelled converter as well as the non-linear characteristic of the magnetic material used. Thus, it leads to accurate results with regard to the real system. Then, it can be helpful to determine the electrical characteristics of a PMLSM when it is difficult to measure them. It can also be used, in virtual prototypes, to quantify their electromagnetic variables in order to foresee the control of the machine or even to modify the geometrical characteristics of the prototype to yield the required performance.

In this paper, we propose to use the finite-element method to study the non-sinusoidal emf of a PMLSM.

In the first part, we introduce the finite-element method and the two potential formulations used to express and solve the electromagnetic problem.

The second part of the paper is devoted to the presentation of the studied system. We show the elementary part modelled and the geometrical characteristics whose effects are studied. The results obtained by both formulations are given in different cases. Comparisons are carried out on the accuracy and time calculations of both formulations. The effect of different geometrical characteristics, studied by the FEM approach, are shown.

In the last part, the simulation results are compared to those obtained on the experimental bench. The studied system is a prototype designed for high speed laser cutting (Fig. 1). To reach high acceleration and velocity, two linear motors from Siemens (LIMES400/120-P16) are used and controlled in parallel.

2. The Finite-Element Method

The finite-element method is used to solve the Maxwell equations related to three or two dimensional electromagnetic converter problems.

2.1 Formulations in term of potentials In the magnetostatic case, the Maxwell equations are written, on a domain “V” of boundary Γ, such as Γh ∩ Γb = Γ and Γh ∪ Γb = 0, under the following form:
\textbf{curl} \mathbf{h} = \mathbf{j}_s \text{ with } \mathbf{h} \times \mathbf{n} = 0 \text{ on } \Gamma_b \hspace{1cm} (1)

\text{div} \mathbf{b} = 0 \text{ with } \mathbf{b} \cdot \mathbf{n} = 0 \text{ on } \Gamma_b \hspace{1cm} (2)

In these expressions, \( \mathbf{h} \) represents the magnetic field, \( \mathbf{b} \) the magnetic flux density and \( \mathbf{j}_s \) the current density. Vector fields \( \mathbf{b} \) and \( \mathbf{h} \) are linked by the constitutive relationship \( \mathbf{b} = \mu \mathbf{h} \). In the case of permanent magnets, the constitutive relationship takes the form:

\[ \mathbf{b} = \mu_a (\mathbf{h} + \mathbf{h}_c) \hspace{1cm} (3) \]

where \( \mathbf{h}_c \) is the coercitive field and \( \mu_a \) is the permeability of the permanent magnets, which is close to the permeability of the air.

In our development, the studied domain “\( V \)” is considered simply connected. To solve these equations, we can use a formulation in terms of scalar potential \( \phi \) (called \( \phi \)-formulation) with a source field \( \mathbf{h}_s \) or a formulation in terms of magnetic vector potential \( \mathbf{a} \) (called \( a \)-formulation).

In the case of the \( \phi \)-formulation, the current density is expressed from the curl of a magnetic field, as:

\[ \mathbf{h} = \mathbf{h}_s - \text{grad} \phi \text{ with } \mathbf{h}_s \times \mathbf{n} = 0 \text{ on } \Gamma_b \hspace{1cm} (4) \]

The magnetic scalar potential \( \phi \) is then the unknown and the source field \( \mathbf{h}_s \) is defined by:

\[ \text{curl} \mathbf{a} = \mathbf{j}_s \hspace{1cm} (5) \]

The formulation in terms of vector potential is obtained from equation (2) so that:

\[ \text{curl} \mathbf{a} = \mathbf{b} \hspace{1cm} (6) \]

To impose the uniqueness of \( \mathbf{a} \), it is necessary to add a gauge condition.

To take into account the non-linear behaviour of the magnetic circuit, we have used an analytical approximation of the non linear curve given by \( \mathbf{h}(\mathbf{b}) \):

\[ \mathbf{h} = \frac{\mathbf{b} \varepsilon + \epsilon \mathbf{b}^2}{\mu_0 + \mu_\tau \mathbf{b}^2} \hspace{1cm} (7) \]

Where \( \mu_0 \) is vacuum permeability and \( \mu_\tau \) is relative permeability. In the linear case: \( \mu_\tau \text{= cest} \). In the saturated case: \( \mu_\tau \) is deduced from the nonlinear magnetic curve \( \mathbf{b}(\mathbf{h}) \) of the ferromagnetic sheets (Fig. 2).

\[ \Delta \]

The differential equation system is solved using an implicit Euler algorithm taking a constant time interval while considering the magnetic material non linearity using an iterative procedure (Newton-Raphson).

2.2 \textbf{Flux linkage expressions} We assume that the domain “\( V \)” has one inductor and we denote “\( I \)” the current. We also note that the current density distribution is assumed to be uniform in each cross section. Consequently, \( \mathbf{j}_s \) in the inductor can be written as:

\[ \mathbf{j}_s = N \mathbf{i} \hspace{1cm} (8) \]

where \( N \) is the vector of turn density. Its magnitude is given by the ratio of the number of turns to the winding section and its direction by its spatial orientation. The flux linkage in a winding can be obtained by integrating, on the whole domain, the projection of the magnetic vector potential on the vector of turn density. Thus, we can write:

\[ \Phi = \int_V \mathbf{a} \cdot \mathbf{N} \text{dv} \hspace{1cm} (9) \]

To determine flux linkage in the case of the \( \phi \)-formulation, as the current density vector \( \mathbf{N} \) is divergence free, a vector \( \mathbf{K} \) is introduced under the form: \( \text{curl} \mathbf{K} = \mathbf{N} \) \footnote{3}. Then, from equation (9), permuting \( \mathbf{N} \) with its expression in function of \( \mathbf{K} \) and using Green’s formula, we can write:

\[ \Phi_w = \int_V \mathbf{K} \text{curl} \mathbf{a} \text{dv} \hspace{1cm} (10) \]

From equations (10) and (6) and considering the constitutive relationship and the expression of magnetic field (3), we can write the flux in function of \( \mathbf{K} \) and \( \phi \) under the form:

\[ \Phi_w = \int_V \mu \mathbf{K} \cdot (\mathbf{K} - \text{grad} \phi) \text{dv} \hspace{1cm} (11) \]

Equations (9) and (11) permit to calculate the flux in the winding using both classical formulations.

3. \textbf{The studied system} The machine being studied is a LIMES400/120-P16, a PMLSM of 6.6 kN, 45 A, and 1.67 m/s. The double-layer armature windings, with eight pairs of poles, are disposed in 51 slots.

Previous studies have shown that the high number of poles decreases the end effects and the dissymetry of the inductances \footnote{6,7}. Consequently, it is possible to use the geometric periodicity to model only a part of the machine and then to limit the computation time without any effect on the accuracy of the results. In figure 3, we show the mesh of the model part.

![Fig. 3. Mesh of a pole of LIMES400/120](image)

The 3D mesh used is made up of 35764 elements, 27141 nodes, 98878 edges and 107502 facets.
4. FEM results

The calculations using 3D-FEM concern the study of the effects of the magnetic material non-linearity and the one of the pole shoe using both formulations. Thus, the following cases have been investigated:
- PMLSM without pole shoe (No PS),
- PMLSM with pole shoe (PS).
- Linear ferromagnetic material (Lin),
- Saturated ferromagnetic material (Sat).

4.1 Comparison $\phi$-formulation / $a$-formulation

The first calculations deal with the emf coefficient in the different cases. Thus, the following comparisons are performed:
- Formulation: $\phi$-formulation or $a$-formulation.
- Speed: V = 1m/s.
- Air-gap: $g = 2.2mm$.
- Magnet induction used in calculations is $\sim 1.2$ Tesla.
- Air-gap surface: $S = 75\%$.

The first step consists of presenting a comparison of the emf coefficient harmonics given by both formulations in linear and non-linear cases.

Table 1. Emf $E_{an}$ rms value (V) for a velocity of 1m/s

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>No PS, Lin</th>
<th>PS, Lin</th>
<th>No PS, Sat</th>
<th>PS, Sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Harmonic $\phi$-formulation</td>
<td>38.08</td>
<td>40.37</td>
<td>39.06</td>
<td>40.91</td>
</tr>
<tr>
<td>1st Harmonic $a$-formulation</td>
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<td>41.10</td>
<td>38.24</td>
<td>40.04</td>
</tr>
<tr>
<td>3rd Harmonic $\phi$-formulation</td>
<td>37.42</td>
<td>39.63</td>
<td>38.25</td>
<td>39.88</td>
</tr>
<tr>
<td>3rd Harmonic $a$-formulation</td>
<td>38.09</td>
<td>40.35</td>
<td>37.41</td>
<td>38.99</td>
</tr>
<tr>
<td>5th Harmonic $\phi$-formulation</td>
<td>6.92</td>
<td>7.59</td>
<td>7.68</td>
<td>8.83</td>
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<tr>
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<td>7.69</td>
<td>7.81</td>
<td>8.89</td>
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<tr>
<td>7th Harmonic $\phi$-formulation</td>
<td>1.18</td>
<td>1.21</td>
<td>1.32</td>
<td>1.50</td>
</tr>
<tr>
<td>7th Harmonic $a$-formulation</td>
<td>1.14</td>
<td>1.20</td>
<td>1.44</td>
<td>2.00</td>
</tr>
</tbody>
</table>

We notice that the $a$-formulation always gives greater values than the $\phi$-formulation. Moreover, it can be noted that whatever the formulation used, the coefficients are greater with the pole shoes.

4.2 Comparison linear / saturated

In Table 2, we present a comparison of the emf coefficient harmonics given by both formulations in linear and non-linear cases.

Table 2. Peak value of emf harmonic $E_{an}$ (V) for a speed of 1m/s

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>No PS, Lin</th>
<th>PS, Lin</th>
<th>No PS, Sat</th>
<th>PS, Sat</th>
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<td>39.63</td>
<td>38.25</td>
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<td>7.69</td>
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<tr>
<td>5th Harmonic $a$-formulation</td>
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<td>1.20</td>
<td>1.44</td>
<td>2.00</td>
</tr>
</tbody>
</table>

In both formulations, the emf values are very close: about 5%. Indeed, the air-gap is about 2.2mm and the magnet induction used in calculations is about 1.2 Tesla. This induces a low level of saturation in the magnetic circuit of the primary, which implies that emf values obtained in this case are lower than those obtained with a linear ferromagnetic circuit. The emf in different cases are shown in the Fig. 4.

4.3 Influence of Pole Shoes

The results on emf coefficient calculations with the $\phi$-formulation and the $a$-formulation are compared in Table 1 and Table 2. Between the PMLSM without pole shoes and the PMLSM with pole shoe, we notice that emf values increase by about 5%.

Indeed, the pole shoes increase the surface of steel in front of the air-gap. Here, the air-gap surface increases from 50% to 75% of the maximum air-gap surface. Thus, the magnetic flux induced in a phase is increased. The main interest of pole shoes PMLSM is that 5 extra percent of emf values results in more generated electromagnetic thrust, but reduces power factor. But, at the same time, the pole shoes increase the value of emf harmonics, which generate ripple force.

4.4 Computation Time

The Maxwell equations have been solved on a Cluster PC XEON microprocessor with 2.8GHz frequency and 512 Mb RAMBUS PC1066 (533MHz). The required degree of accuracy for both formulations is fixed to $10^{-12}$ in the numerical resolution of magnetic flux density. $\phi$-formulation and $a$-formulation computation times are compared in Table 3.

Table 3. Computation Time (min)

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>No PS, Lin</th>
<th>PS, Lin</th>
<th>No PS, Sat</th>
<th>PS, Sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Harmonic $\phi$-formulation</td>
<td>26</td>
<td>26</td>
<td>1302</td>
<td>1339</td>
</tr>
<tr>
<td>1st Harmonic $a$-formulation</td>
<td>38</td>
<td>38</td>
<td>315</td>
<td>455</td>
</tr>
</tbody>
</table>

For the analysis of the linear ferromagnetic material, the $a$-formulation needs more computation time than the $\phi$-formulation. In fact, the numerical resolution of the Maxwell equations is realised with a large amount of matrix “juggling”, which represents the interaction between the magnetic state of each element of the system. So, the complexity of the system can be seen with the matrix size. Here, we have a 365771 matrix size for the $\phi$-formulation and a 232682 matrix size for the $a$-formulation. Hence, the computation time ratio between the two formulations is very close to the matrix size ratio of about 1.5.

For the analysis of the saturated ferromagnetic material, the $a$-formulation needs more computation time than the $\phi$-formulation. Indeed, our iteration procedure of the $a$-formulation is based on the Newton-Raphton, whereas the $\phi$-formulation is based on the substitution algorithm. Classically, the $a$-formulation needs three times more iteration for a similar position analysis.

4.5 Conclusion of FEM analysis

Regarding to results in Table 1, Table 2 and Table 3, in studied cases, the $a$-formulation requires too much computation time compared to the $\phi$-formulation. Furthermore, the emf values reached with each formulation are very close. And finally, the level of saturation in the system is not enough to justify the computation time required in the saturated case.

The emf analysis of the PMLSM in the linear case, with pole shoes and in the $\phi$-formulation, requires an acceptable computation time, and gives precise enough results on the emf values (less than 10% difference with other results) regarding all the hypotheses taken. The values of the emf harmonics obtained by FEM are shown in Table 4.
Table 4. Harmonics of emf by FEM (V)

<table>
<thead>
<tr>
<th></th>
<th>Rms Ean</th>
<th>H1 Rms</th>
<th>H3/H1</th>
<th>H5/H1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rms H1</td>
<td>40.37</td>
<td>39.63</td>
<td>19%</td>
<td>3%</td>
</tr>
</tbody>
</table>

5. Experimental Validation

Experimentally, a special technique is required to identify the harmonic components of the emf (Fig. 6). Indeed, steady state at a rated speed is difficult to reach because of the finite-length of the PMLSM. Furthermore, the PMLSM coils are star-connected, which eliminates the 3rd harmonics of the line-to-line emf. The induced voltage is divided by the instantaneous speed to give the emf. The quality of the results depends on the high precision of the position measurement. Here, we have used a Heindenhain exposed linear encoder with a grating period of 40µm.

![Fig. 6. Experimental results of emf](image)

We obtain from FEM: a simple emf \( E_{an} \) and a line-to-line emf \( E_{ab} \) (Fig. 6). With a star connected primary, we confirm that the 3\(^{rd} \) harmonic has disappeared.

![Fig. 7. FEM results of simple emf and line-to-line emf](image)

![Fig. 8. experimental results are compared to the FEM results.](image)

![Fig. 8. Comparison FEM / experimental results of emf](image)

In order to simplify the meshing of the PMLSM, and to reduce the computation time, we have neglected the end winding effects and the end longitudinal effects. Consequently, the FEM results underrated the emf values.

6. Conclusion

In this paper, we have presented the finite-element analysis of non sinusoidal electromotive force of a permanent magnet linear synchronous motor. We have verified on the emf rms value that FEM and experimental results are close (the maximal error is about 5% of the emf fundamental values). That allows us to validate the approach and to understand the effects of geometrical characteristics. Then, a notable advantage of FEM analysis is to obtain the third harmonic of emf, whereas experimental results cannot give this result, because the PMLSM phases are star-connected, with an inaccessible neutral wire. As we have neglected the coils end effects and the longitudinal end effects, we cannot determine precisely the fifth and upper level harmonic by FEM. On the other hand, experimental results give harmonic values of emf up to the 7\(^{th} \) rank with a variance of less than 10% (out of more than thirty analyses).

Several cases have been studied and analysed: with or without pole shoes, in linear or saturated ferromagnetic material, with \( \phi \)-formulation or \( a \)-formulation.

For high-precision applications, the optimal control of the position for a system is firstly based on the control of the PMLSM. Non-sinusoidal electromotive forces generate ripple force on the electromagnetic thrust. Resonant controllers allow us to compensate harmonics of emf, and so to generate a non-oscillating electromagnetic thrust. (8)

References

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