Finite Element Analysis of a PMLSM (part 1)
- Meshing techniques and thrust computations -

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1. Introduction

Nowadays, numerical approaches are widely used to model electrical machines. Among them, the 2D-finite-element method (2D-FEM) is the most frequently encountered to study the behaviour of electromagnetic devices [1]. Indeed, it takes into account both the geometry of the system and the non linearity of the magnetic material. Then, it offers accurate results with regard to the real system. To obtain fast optimizations of electric machine designs, a low number of mesh elements is required in order to limit the computation time. However, this generally underruns the accuracy of the results. In the case of global electrical characteristics such as fluxes or electromotive forces, the quality of the mesh does not affect the result greatly. This is not the case of the no-load torque or force. Indeed, accurate calculations are highly dependent on the method used to perform them and on the mesh quality of the modelled part of the device. Hence, one has to take care of these parameters to reach useful results, especially given that the no-load torque or force are generally very difficult to obtain from measurements. These difficulties are increased in the case of the permanent magnet linear synchronous machines (PMLSM) as the no load force is composed of two parts, the cogging and the extremity ones [2].

In this paper, in order to accurately calculate the no-load force using the FEM, we propose a meshing procedure of a PMLSM and a comparison of force formulations.

In the first part, we introduce the formulations that are generally used to calculate the forces in the FEM codes. The problem of the force accuracy calculation induced by the meshing size is considered.

In the second part of the paper is devoted to the mesh design of a PMLSM. We define an elementary part of the PMLSM to be modelled and the main geometrical characteristics are explained. The results obtained from different meshes are analyzed. Some comparisons are carried out on the effectiveness of force formulations, the accuracy of the results and the time calculations.

In the last part, the simulation results are compared to those obtained on an experimental bench.

The studied system is a linear motor LMD10-050 (Fig. 1) from the ETEL Company. It is classically used as a component of Pick-and Place systems.

2. The force calculation using the Finite Element Method (FEM)

To calculate the electromagnetic forces in electrical systems, different methods can be used [3]. However, in most of the FEM codes, two approaches are generally used: The Virtual Work Method [4] or the Maxwell Stress Tensor [5].

2.1. The Virtual Work Method

This approach is based on the transformation of magnetic energy to mechanical energy. We can show that the total force in one direction $s$ is calculated from the magnetic energy variation $W$ of the system after a displacement in this direction. The motion is done at constant flux [6]-[8].

$$ F_s = -\frac{\partial W}{\partial s} \bigg| \Phi = \text{cte}, \text{ with } W = \int_0^1 \Phi \, dh \, dv $$

A similar expression can be deduced from the variation of the co-energy at a constant current.

$$ F_i = \frac{\partial W'}{\partial i} \bigg| i = \text{cte}, \text{ with } W' = \int_0^1 \Phi \, dh \, dv $$

Different techniques can be used to compute the force:
- A 1st method requires two solutions of the problem and even if a very fine mesh is not needed, numerical errors can appear if the displacement step (for the calculation of the derivate) is too high.
- A 2nd method is to do only one computation of the problem. Having the different fields on each element, then the magnetic energy is calculated for two positions of an object.
- Similarly, a 3rd method consists in taking an air layer surrounding an object. The nodes of the layer are then virtually displaced. For each deformation the energy is calculated.

2.2. The Maxwell Stress Tensor

In the case of the Maxwell Stress Tensor (MST), we have to calculate the divergence of the following tensor:

$$ T_{ij} = \mu_0 (H_i H_j - \frac{1}{2} \delta_{ij} H^2) \text{ with } i, j = x, y, z $$

With $H$ the magnetic field given by its values in the Cartesian frame $(x,y,z)$ $H(x, H, H, H)$. $\delta_{ij}$ is the Kronecker sign ($\delta_{ij} = 1$ if $i=j$ otherwise $\delta_{ij} = 0$).

The Maxwell Stress Tensor is performed by a
calculation of the force using a surface integration on \( \Gamma' \), over a \( D' \) domain. The force can be obtained using the following formula:

\[
F = \iint_D \text{div} \mathbf{D} \, d\mathbf{V} = \iint_D \left( \mathbf{H} \cdot \mathbf{n} \right) \, d\mathbf{S} - \frac{1}{2} \iint_D \left| \mathbf{B} \right|^2 \cdot \mathbf{n} \, d\mathbf{S} \quad (4)
\]

The vector \( \mathbf{n} \) is the normal on the surface \( \Gamma' \). The Maxwell Stress Tensor presents several advantages, linear or non linear cases can be evaluated, the choice of the surface integration is undistinguished and only one part of the mesh is concerned.

However, this method is very sensitive to the mesh quality and density of the region where the surface of integration is taken. It is then preferable to have enough air layers on the \( D' \) domain surrounding the objects on which the forces have to be calculated. We noticed that acceptable results were obtainable with at least 4 to 5 air layers. This is particularly the case in the air-gap areas, where it usually has the higher field changes.

The Maxwell Stress Tensor is based on the calculation of the divergence of the Maxwell Stress Tensor \([9]\). And the classical formulation detailed in Eq. 4 induces that the permeability has to be taken constant equal to the air permeability. So the force calculation has to be done in the region surrounding the studied object.

### 2.3. Other methods

Added to the VWM and the MST, other methods are used, less frequently, in FE codes to calculate the force. We can quote the eggshell method \([5]\), the equivalent magnetizing current for the force calculation \([10]\) and the equivalent magnetic mass method \([11]\).

### 2.4. Typical problems encountered using the force calculations

When calculating the forces, often some accuracy issues occur at the no-load operating point. The most common is the no-load force offset. The meshing size of the system is generally a predominant factor, as it influences the numerical errors directly.

### 3. Meshing techniques for the force calculation

#### 3.1. Presentation of the studied PMLSM

This PMLSM is constituted of a teethed ironcore short primary with an armature of three phase concentrated windings. The secondary is composed of a set of alternating Nd-Fe-B magnets, with a residual flux density \( B_r = 1.23 \) T. Table 1 gives the main dimensions and specifications of the PMLSM studied. Figure 2 shows a cutting view of the whole studied PMLSM:

![Figure 2: Cutting view of the whole PMLSM [12].](image)

Besides, the no-load forces along the motion direction are generally very low when compared to attractive forces (in the case of the studied PMLSM, attractive forces are about 1770N while the no-load detent force is estimated at 4N). This offset is particularly visible for the flat ironcore linear motor because of the huge difference between the attractive force and the detent force.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value [Unit]</th>
</tr>
</thead>
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<tr>
<td>Number of poles</td>
<td>2</td>
</tr>
<tr>
<td>Residual induction</td>
<td>1.23 [T]</td>
</tr>
<tr>
<td>Height of PM</td>
<td>4 [mm]</td>
</tr>
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<td>Length of PM</td>
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<td>14 [mm]</td>
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<td>Pole pitch</td>
<td>16 [mm]</td>
</tr>
<tr>
<td>Length of air-gap</td>
<td>0.8 [mm]</td>
</tr>
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</table>

Table 1: Dimension and specification of analysis model

Figure 3 presents FEM results of the detent force. The mesh has 26525 Nodes, 52592 Elements and gives for each step 2 min of computation time in the linear case. The detent force is calculated with a large meshing size, using the MST formulation. Details of the meshing size are given in the first line of Table 2. The full geometry is detailed in the next paragraph.

![Figure 3: Offset on the force calculation](image)

#### Figure 3: Offset on the force calculation

### 3.2. Geometry and mesh methodology

Here, we briefly propose a mesh procedure that could lead to an accurate force calculation: since much direction fluctuation of the flux density happens in the air-gap, it is necessary to represent this active zone with at least three layouts. So the meshing size has to be chosen first in the air-gap, and then it can be larger everywhere else. Furthermore, for the force calculation, a fine cover of meshed air greatly improves the results. That imitates a technique called the eggshell method \([5]\). Figure 4 shows a mesh designed for the actuator (LMD10-050):

![Figure 4: Geometry with mesh size of 0.3 mm.](image)

The added numbers on Figure 4 correspond to zones with different mesh sizes. The rule is to mesh only the zones which are under fast changes of the flux.

Table 2 shows results of the detent force using the MST and the VWM for different mesh sizes of the geometry.

Figure 5 shows a zoom of Figure 4. We can notice in Zone 4, called the air eggshell, that a 0.3mm (compared to 0.8mm of the air gap) mesh is particularly efficient. This
air eggshell acts as a cover on the primary ironcore, and permits to follow the curve of the magnet induced flux more accurately. The other zones are defined with a larger mesh size of 0.8mm, with an auto-adaptation of the mesh to connect with the finer meshed Zone 4 of the air eggshell.

Figure 5: Zoom of Figure 4.

About the force formulations for the Maxwell Stress Tensor, the calculation uses an integral on the volume of the laminated steel of the primary. Calculation results are also closed by choosing the volume of the secondary (including the yoke and all the magnets). For the Virtual Work Method, the variation of the co-energy is calculated over the whole system for more precise results.

The influence of the meshing size can be directly visible on the value of the detent force. To obtain the offset value, MST calculations can be performed at a position with no detent force, or we can calculate under movement the mean value of the detent force.

We can notice in Table 2 that smaller mesh size is not always the best solution.

### 3.3. Detent force results

To calculate the detent forces, different movement techniques can be used. The results are more accurate with no change in mesh between two steps. Hence, the locked step approach is well adapted. In magnetostatic cases, the motion can also be performed by permuting some materials (the magnets in our study).

Figure 6 shows the nonlinear magnetic curve $b-h$ of the ferromagnetic sheets. Model details are given in [13].

Figure 6: $b-h$ curve of the laminated sheet.

Figure 7 shows the magnetic flux density of Figure 4.

Figure 7: Flux induction field.

Figure 8 shows the no-load force calculated using the MST for a linear and a non-linear case. We can notice that the results are very close. So for minimizing computation time, the linear case is then only studied.

Figure 8: No load force of a LMD10-050 (ETEL).

Figure 9 shows the results of the no-load force calculated by both approaches (VWM and MST) in the linear case: we can notice more harmonics on the waveform calculated by the Virtual Work Method.

Figure 9: Detent force calculations with both methods

Then, a Fast-Fourier Transform (FFT) is applied on the Figure 9 results, and harmonic values of the detent force are presented in Table 3. Results on harmonics $H_1$ and $H_6$ are close to 8% between both formulations.

Table 3: Comparison of harmonics for both formulations.

<table>
<thead>
<tr>
<th>Formulation</th>
<th>$H_1$</th>
<th>$H_2$</th>
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<th>$H_4$</th>
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<tr>
<td>Virtual Work Method (N)</td>
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<td>1.78</td>
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### 3.4. Conclusion on the force calculation

To obtain accurate detent force calculations, meshing techniques, force formulations and movement techniques have to be carefully chosen.

As the VWM formulation is performed using integration over all the primary ironcore rather than its periphery, it is less sensitive to imperfect field path caused by the underestimation of the meshing size. Nevertheless, results are not good enough to neglect offsets on results.

### 4. Experimental validation

The proposed model is experimentally verified on a laboratory test system equipped with an ETEL LMD10-050 linear motor (Fig. 1).

The detent force is estimated using Newton's 2nd law:

$$ M \frac{dv}{dt} = T_{em} - T_{det} - T_{vis} - T_{load} $$

(3)
With $T_{det}$, the detent force, $T_{load}$, the load force, $T_{Coul}$, the Coulomb force; $T_{vis} = f_{vis} \cdot v$, the viscous friction force, $M$, the mass of the mobile, $v$, the primary velocity and $T_{em}$, the electromagnetic thrust.

Figure 10 shows the no-load force waveform measured on the actuator [13]. Some fast force peaks are still visible in Fig. 10, due to the Power Electronic commutations.

5. Conclusion

In this paper, first, we have presented some calculation techniques used in the FEM to calculate the forces. Then, we underline the problems which are generally encountered in the force calculation. The main one is the quality of the mesh. A mesh technique has been introduced in order to reach accurate results for the no load force. This technique has been used to study the behaviour of a PMLSM. Both VWM and MST give good results. The simulation waveform of the no load force have been compared to measurements and show a good agreement.

Acknowledgement

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References


Table 2: Offset value on the detent forces using MST and VWM of Figure 4.

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<tr>
<th>1(mm)</th>
<th>2(mm)</th>
<th>3(mm)</th>
<th>4(mm)</th>
<th>5(mm)</th>
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