

The decidability of the domino problem

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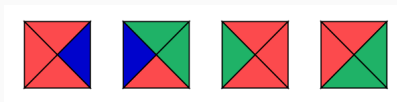
ÉCOLE
POLYTECHNIQUE



Introduction

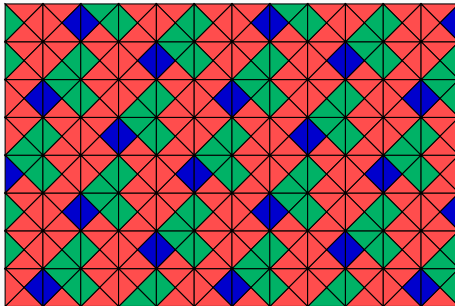
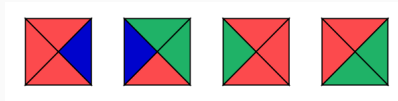
Wang Tileset

- A set of tiles with compatibility rules



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Theorem (Easy inclusion, compacity argument)

*Given a set of tiles, the problem of knowing whether it paves the plan is *co-computably enumerable*.*

False Conjecture (Wang '61) : all the paving of the plan are *periodic*.

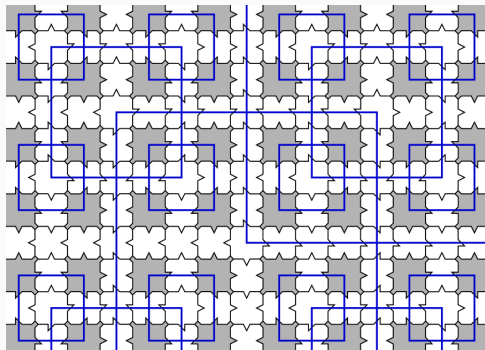
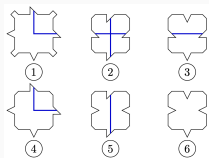
Theorem (Easy inclusion, compacity argument)

Given a set of tiles, the problem of knowing whether it paves the plan is co-computably enumerable.

False Conjecture (Wang '61) : all the paving of the plan are periodic.

FALSE!!

Example : Robinson tileset '78

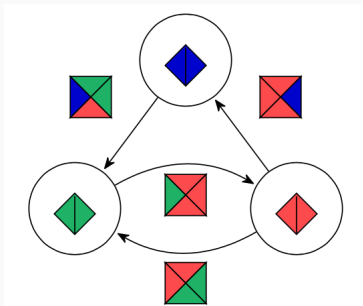


Theorem (Berger '64)

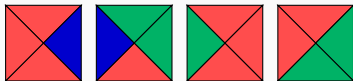
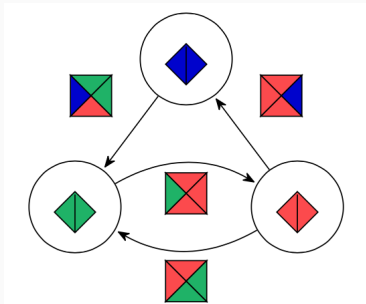
*Given a set of tiles, the problem of knowing whether it paves the plane is **undecidable**.*

The formalism of transducers

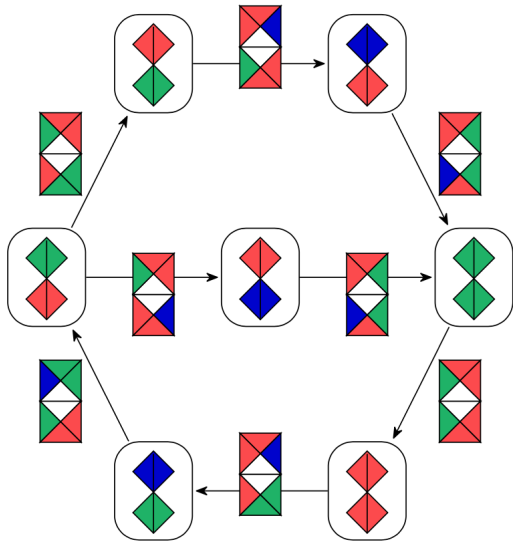
Transducers



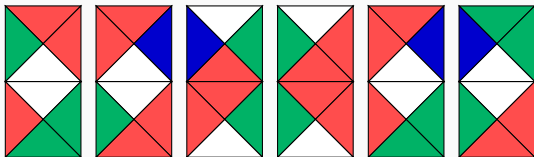
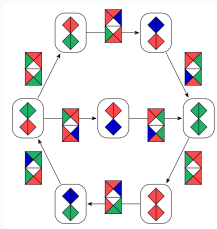
Example



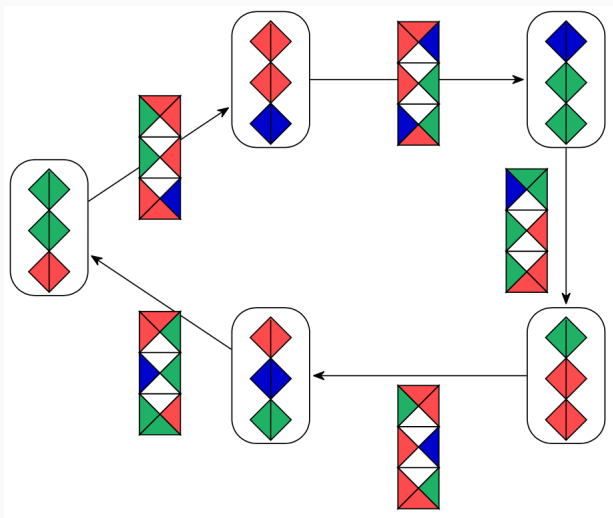
(Meta-)Transducers



Example



(Meta-)Transducers



The notion of robustness

Definition (Robustness)

Let τ a tileset. It is *provably robust* if there *provably* exists some sequence of transducers $(\mathcal{T}_n)_{n \in \mathbb{N}}$ such that, for all $n \in \mathbb{N}$, \mathcal{T}_n contains a loop

Theorem

Let τ a tileset. τ is *provably robust* $\Leftrightarrow \tau$ admits a tiling.

→ the domino problem for provably robust tileset is computable

Conclusion

Conclusion

- We have a criterion to prove if a tileset can tile a plan.
- We applied it to some well-known tilesets, including Jeandel-Rao 2015 tileset and Robinson 1978 tileset.
- Current work : a criterion to prove if a tileset can *aperiodically* tile.