

Computability and Complexity

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Who the hell am I?

Computability: What is a Turing machine

Space complexity vs. Time complexity

Definition (Linear length ODE)

Une fonction \mathbf{f} est (resp. linéaire) “length-ODE” définissable (de \mathbf{u} *essentiellement linéaire* en $\mathbf{f}(x, \mathbf{y})$, \mathbf{g} et \mathbf{h}) si cela correspond à la solution de

$$f(0, \mathbf{y}) = \mathbf{g}(\mathbf{y}) \quad \text{and} \quad \frac{\partial \mathbf{f}(x, \mathbf{y})}{\partial \ell} = \mathbf{u}(\mathbf{f}(x, \mathbf{y}), \mathbf{h}(x, \mathbf{y}), x, \mathbf{y}). \quad (1)$$

Fondamentalement, la derivation en la longueur revient à faire un changement de variables:

Lemma

Supposons (1) vraie. Alors, $\mathbf{f}(x, \mathbf{y})$ est donnée par $\mathbf{f}(x, \mathbf{y}) = \mathbf{F}(\ell(x), \mathbf{y})$ où \mathbf{F} est la solution du "initial value problem"

$$\mathbf{F}(1, \mathbf{y}) = \mathbf{g}(\mathbf{y}), \quad \text{and} \quad \frac{\partial \mathbf{F}(t, \mathbf{y})}{\partial t} = \mathbf{u}(\mathbf{F}(t, \mathbf{y}), \mathbf{h}(2^t - 1, \mathbf{y}), 2^t - 1, \mathbf{y}). \quad (2)$$

Inspiré par:

$$\frac{\delta f(x, \mathbf{y})}{\delta x} = \frac{\delta \mathcal{L}(x, \mathbf{y})}{\delta x} \cdot \frac{\delta f(x, \mathbf{y})}{\delta \mathcal{L}(x, \mathbf{y})}.$$

la fonction varie seulement quand:

$$\mathcal{L}(x + 1, \mathbf{y}) - \mathcal{L}(x, \mathbf{y}) \neq 0$$

Exemple de length-ODE

$$f(0) = 2$$

$$\frac{\partial f}{\partial \ell}(x) = f(x) \cdot f(x) - f(x)$$

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Unique solution : $f(x) = 2^{2^{\ell(x)}}$

Somewhere over the real numbers

$\text{LDL}^\bullet =$

$[\mathbf{0}, \mathbf{1}, \pi_i^k, \ell(x), +, -, \times, \text{cond}, \frac{x}{2}; \text{composition, linear length ODE}]$

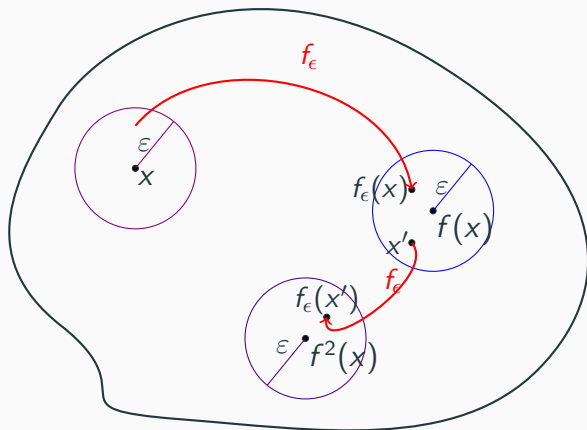
$\text{LDL}^\bullet \cap \mathbb{R}^{\mathbb{N}} = \text{FPTIME} \cap \mathbb{R}^{\mathbb{N}}$

$\text{LDL}^\circ =$

$[\mathbf{0}, \mathbf{1}, \pi_i^k, \ell(x), +, -, \tanh, \frac{x}{2}, \frac{x}{3}; \text{composition, linear length ODE}],$

$\text{LDL}^\circ \cap \mathbb{R}^{\mathbb{R}} = \text{FPTIME} \cap \mathbb{R}^{\mathbb{R}}$

Shake that



What about GaLAC?
