

```
In[1]:= $Assumptions = c > 0 && c < 1 && ρ > 0 && ρ < 1
```

```
Out[1]= c > 0 && c < 1 && ρ > 0 && ρ < 1
```

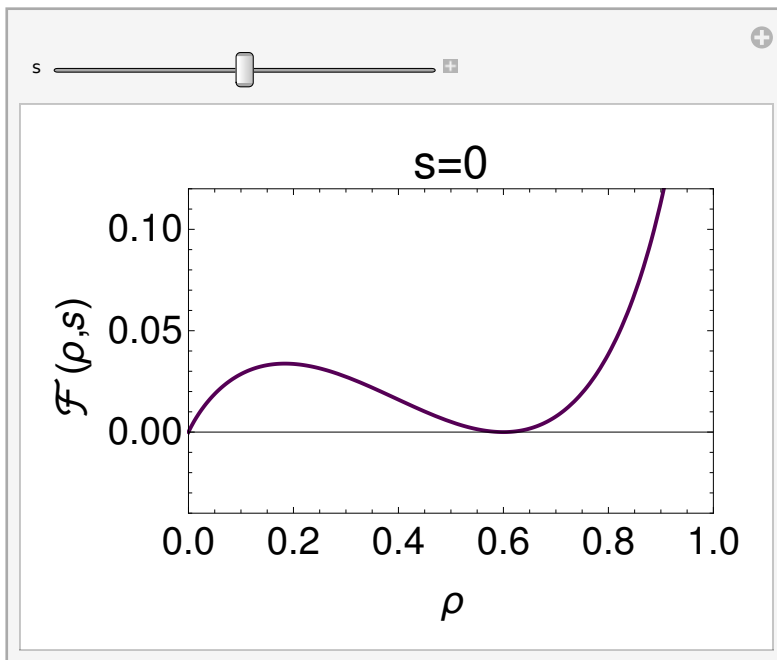
```
In[2]:= (*dynamical Landau free-  
energy for a KCM on a complete graph s conjugated to activity*)  
F[ρ_, s_, c_] = c ρ (1 - ρ) + (1 - c) ρ^2 - 2 Exp[-s] Sqrt[c (1 - c) ρ (1 - ρ)] ρ
```

```
Out[2]= c (1 - ρ) ρ + (1 - c) ρ^2 - 2 e-s ρ √(1 - c) c (1 - ρ) ρ
```

```
In[3]:= (*psi_K(s) is given by the minimum of F[ρ, s, c] for ρ on [0, 1]*)
```

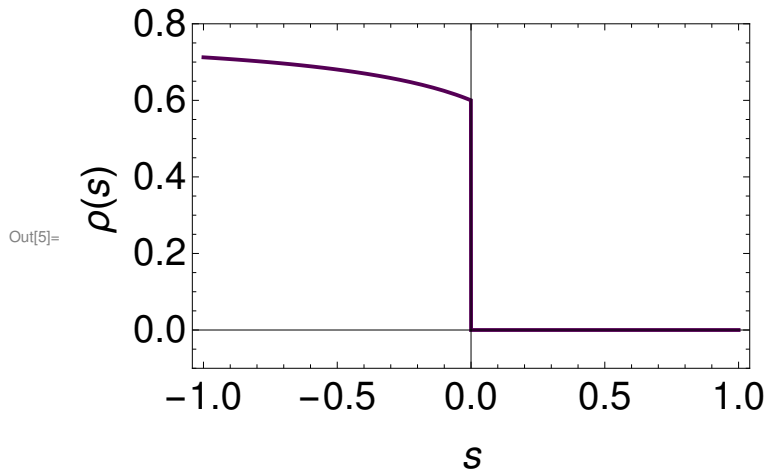
```
Manipulate[  
  cc = .6;  
  Plot[F[ρ, s, cc], {ρ, 0, 1}, PlotStyle → {Thick, Darker@Purple}, Frame → True,  
    PlotRange → {{0, 1}, {-0.04, .12}}, FrameLabel → {"ρ", "F(ρ, s)"},  
    LabelStyle → Directive[FontSize → 20], PlotLabel → "s=" <> ToString[N[s, 3]],  
    {{s, 0}, -.2, .2}]
```

```
Out[3]=
```



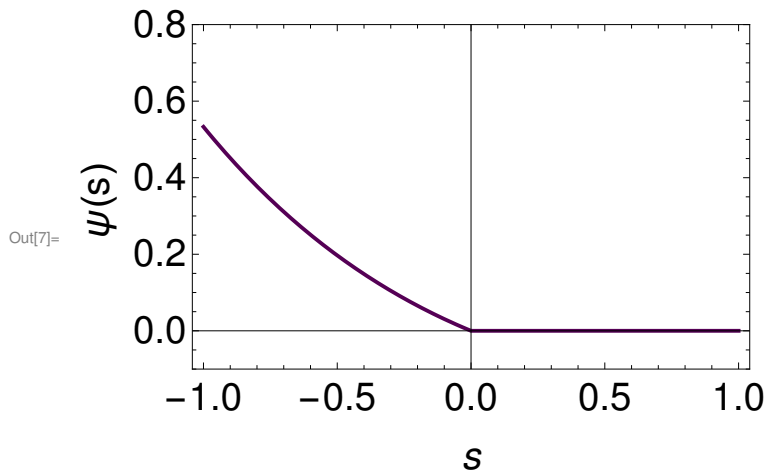
```
In[4]:= pofs[s_, c_] = Simplify [ρ /. Solve[D[F[ρ, s, c], ρ] == 0, ρ][[3]]];
```

```
In[5]:= Plot[Piecewise[{{ $\rho$ ofs[s, cc], s ≤ 0}, {0, s ≥ 0}}, {s, -1, 1}, AxesOrigin→{0, 0},
  PlotStyle→{Thick, Darker@Purple}, Frame → True, Exclusions→False,
  PlotRange→{All, {-0.1, .8}}, FrameLabel → {"s", " $\rho(s)$ "},
  LabelStyle→Directive[FontSize→20]]
```



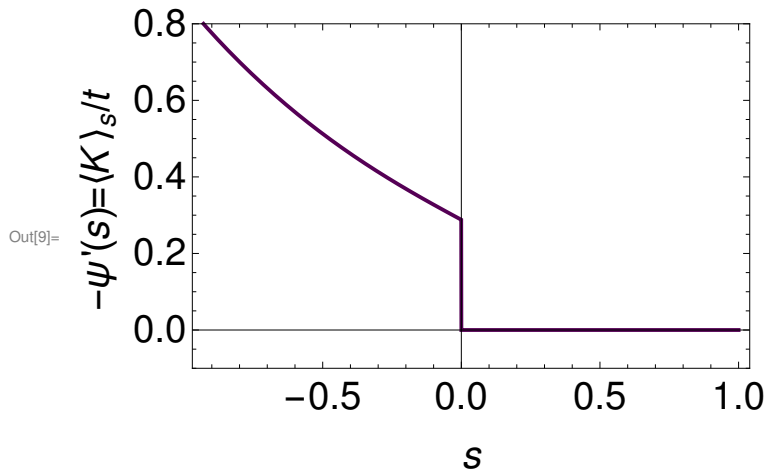
```
In[6]:=  $\psi$ sol[s_, c_] = -F[ $\rho$ ofs[s, c], s, c];
```

```
In[7]:= Plot[Piecewise[{{ $\psi$ sol[s, cc], s ≤ 0}, {0, s ≥ 0}}, {s, -1, 1}, AxesOrigin→{0, 0},
  PlotStyle→{Thick, Darker@Purple}, Frame → True, Exclusions→False,
  PlotRange→{All, {-0.1, .8}}, FrameLabel → {"s", " $\psi(s)$ "},
  LabelStyle→Directive[FontSize→20]]
```



```
In[8]:=  $\psi$ 1sol[s_, c_] = D[ $\psi$ sol[s, c], s];
```

```
In[9]:= Plot[Piecewise[{{-ψ1sol[s, cc], s ≤ 0}, {0, s ≥ 0}], {s, -1, 1}, AxesOrigin → {0, 0},
  PlotStyle → {Thick, Darker@Purple}, Frame → True, Exclusions → False,
  PlotRange → {All, {-0.1, .8}}, FrameLabel → {"s", "-ψ'(s) = ⟨K⟩_s / t"},
  LabelStyle → Directive[FontSize → 20]]
```



In[10]=

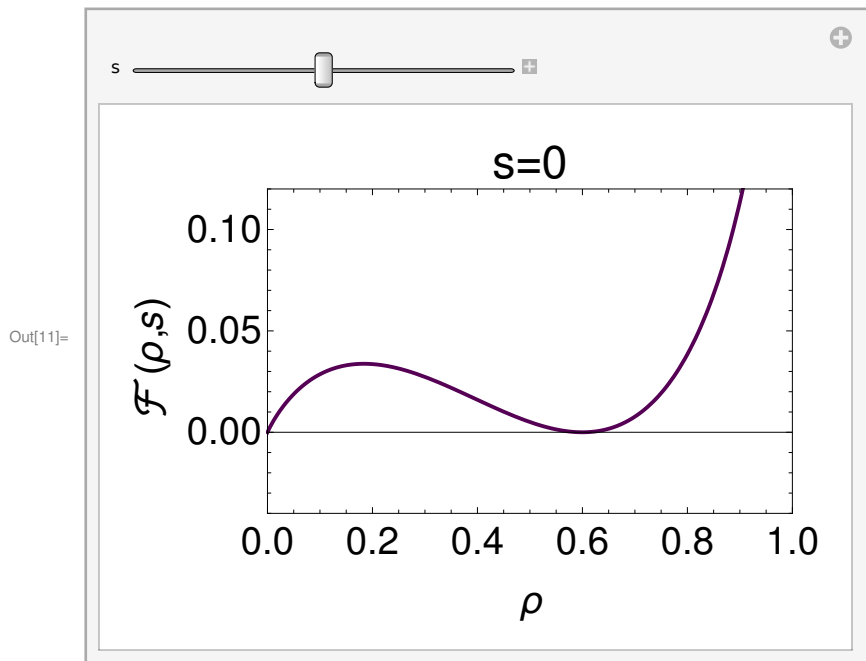
```
In[10]:= (*dynamical Landau free-
  energy for a KCM on a complete graph s conjugated to density*)
Fn[ρ_, s_, c_] = c ρ (1-ρ) + (1-c) ρ^2 - 2 Sqrt[c (1-c) ρ (1-ρ)] ρ + s ρ
```

Out[10]= $s \rho + c (1-\rho) \rho + (1-c) \rho^2 - 2 \rho \sqrt{(1-c) c (1-\rho) \rho}$

```

In[11]:= (*psi_K(s) is given by the minimum of F[ρ,s,c] for ρ on [0,1]*)
Manipulate[
  cc = .6;
  Plot[Fn[ρ, s, cc], {ρ, 0, 1}, PlotStyle → {Thick, Darker@Purple}, Frame → True,
    PlotRange → {{0, 1}, {-0.04, .12}}, FrameLabel → {"ρ", "F(ρ, s)"},
    LabelStyle → Directive[FontSize → 20], PlotLabel → "s=" <> ToString[N[s, 3]],
    {{s, 0}, -.2, .2}]

```



```

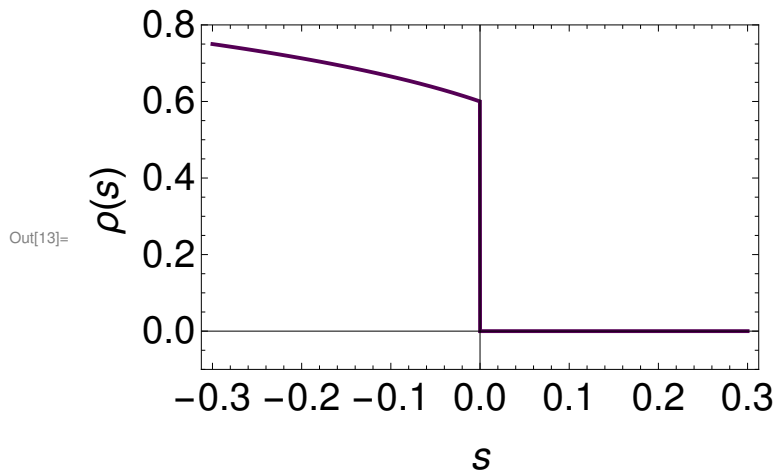
In[12]:= ρnofs[s_, c_] = Simplify [ρ /. Solve[FullSimplify [D[Fn[ρ, s, c], ρ]] == 0, ρ][[3]]];

```

```

In[13]:= Plot[Piecewise[{{ρnofs[s, cc], s ≤ 0}, {0, s ≥ 0}], {s, -.3, .3}, AxesOrigin → {0, 0},
  PlotStyle → {Thick, Darker@Purple}, Frame → True, Exclusions → False,
  PlotRange → {All, {-0.1, .8}}, FrameLabel → {"s", "ρ(s)"},
  LabelStyle → Directive[FontSize → 20]]

```

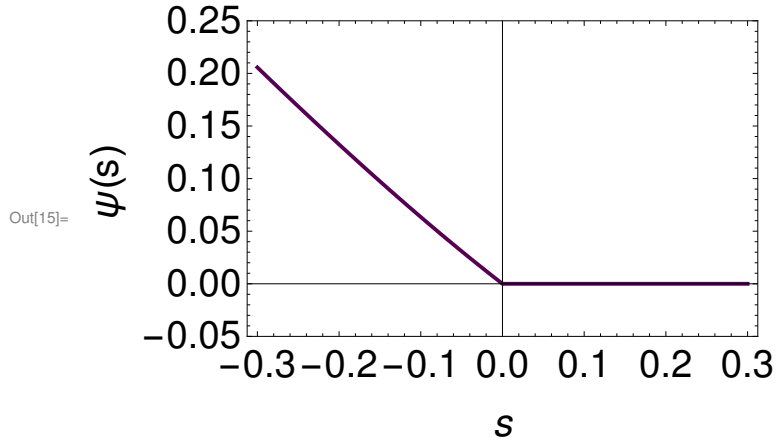


```

In[14]:= ψnsol[s_, c_] = -Fn[ρnofs[s, c], s, c];

```

```
In[15]:= Plot[Piecewise[{{ψnsol[s, cc], s ≤ 0}, {0, s ≥ 0}}, {s, -.3, .3}, AxesOrigin→{0, 0},
  PlotStyle→{Thick, Darker@Purple}, Frame → True, Exclusions→False,
  PlotRange→{All, {-0.05, .25}}, FrameLabel → {"s", "ψ(s)"},
  LabelStyle→Directive[FontSize→20]]
```



```
In[16]:= ψn1sol[s_, c_] = D[ψnsol[s, c], s];
```

```
In[17]:= Plot[Piecewise[{{-ψn1sol[s, cc], s ≤ 0}, {0, s ≥ 0}}, {s, -.3, .3}, AxesOrigin→{0, 0},
  PlotStyle→{Thick, Darker@Purple}, Frame → True, Exclusions→False,
  PlotRange→{All, {-0.1, .8}}, FrameLabel → {"s", "-ψ'(s) = (ρ)_s/t"},
  LabelStyle→Directive[FontSize→20]]
```

