

Alon-Saks-Seymour conjecture, Clique-Stable separation and the stubborn problem

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Geographical context

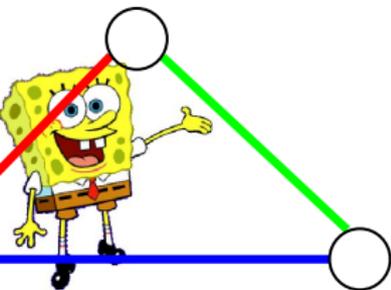


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Clique vs Independent Set Problem



Clique vs Independent Set Problem

Question: Clique-Stable Set separation

Does there exist a polynomial P , such that for all graph G on n vertices, there exists a family of cuts that separates all the cliques and the stable sets (=a Clique-Stable separator) of size $P(n)$?

Bipartite packing $\text{bp}(G)$

Alon-Saks-Seymour conjecture

Alon-Saks-Seymour conjecture (≈ 1974)

If the edges of G can be partitioned into k edge-disjoint complete bipartite graphs, then $\chi \leq k + 1$.

In other words: $\chi \leq \text{bp} - 1$.

Remark

True on complete graphs (Graham-Pollak theorem, 1971).

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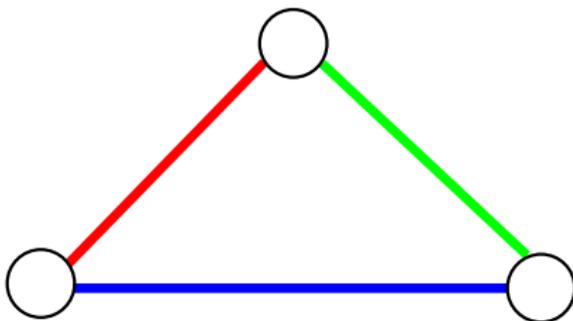
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Huang & Sudakov, 2010: There exists G such that $\chi \geq \text{bp}^{6/5}$.

New question: Polynomial Alon-Saks-Seymour conjecture

Does there exist P such that for all G , $\chi \leq P(\text{bp})$.

3-COMPATIBLE COLORING PROBLEM

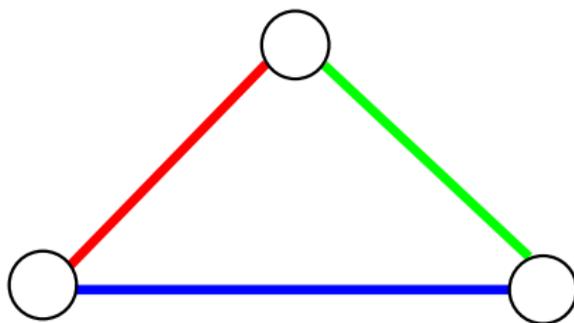


3-COMPATIBLE COLORING PROBLEM (3-CCP)

Input: An edge coloring f_E of the complete graph on n vertices with 3 colors $\{A, B, C\}$.

Question: Is there a coloring of the vertices with the same three colors, such that no edge has the same color as both its extremities?

2-list covering for 3-COMPATIBLE COLORING PROBLEM

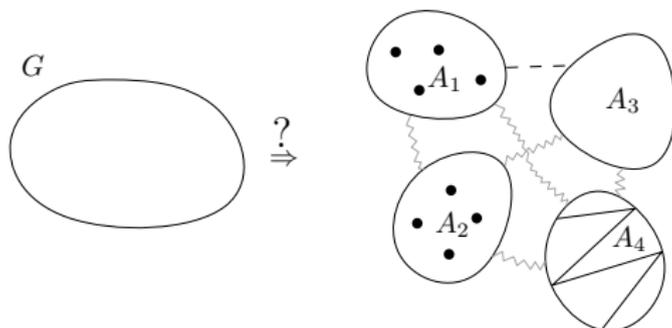


2-list covering for 3-COMPATIBLE COLORING PROBLEM (3-CCP)

Assign to each vertex a list of size 2 compatible with a solution.
Build a family of such objects that covers all the solutions, i.e. for all solution, there is 2-list assignment in my family which is compatible with the solution.

Goal: have a *polynomial* family.

The stubborn problem



The stubborn problem

Input: A graph $G = (V, E)$ together with a list assignment $\mathcal{L} : V \rightarrow \mathcal{P}(\{A_1, A_2, A_3, A_4\})$.

Question: Can V be partitioned into four sets A_1, \dots, A_4 such that A_4 is a clique, both A_1 and A_2 are stable sets, there is no edge between A_1 and A_3 , and each vertex v belongs to A_i only if $A_i \in \mathcal{L}(v)$?

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- Study the Clique-Stable separation on perfect graphs thanks to structure theorem.
- Study which graph operations preserves the property of polynomial Clique-Stable separation.