

Exploring the class XNLP

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Parameterized complexity

Goal Understanding what makes problems tractable/intractable

Tools Algorithmic techniques and simple hardness proofs using reductions

In my work, focus on combinatorial problems.

Basics on parameterized complexity

- ▶ Parameter : a value that is added to the instance, usually a metric on the instance
- ▶ Most common parameter is solution size
- ▶ Reductions must control parameter growth
- ▶ Kernel : reduced equivalent instance of size bounded by the parameter obtained in poly time

Different levels of hardness

From easiest to hardest:

- ▶ Admits polynomial kernel
- ▶ Admits a kernel = FPT: can be solved in time $f(k)n^c$
- ▶ $W[1]$ = reducible to CLIQUE
- ▶ $W[2]$ = reducible to DOMINATING SET
- ▶ XP: can be solved in time $n^{f(k)}$
- ▶ para-NP : is NP-hard for some constant value of the parameter

Common questions

Does my parameterized problem:

- ▶ Admit a polynomial kernel ?
- ▶ Admit an FPT algorithm ?
- ▶ Admit an XP algorithm ?

For each of these questions, there are tools to show negative answers under reasonable assumptions.

What is XNLP ?

XNLP: Class of problems that can be solved nondeterministically in time $f(k)n^c$ and space $f(k) \log(n)$.

These problems can be solved in time $n^{f(k)}$.

Conjecture on XNLP-hard problems:
they cannot be solved in time $n^{f(k)}$ without using $n^{f(k)}$ space.

Relation to other common classes

- ▶ FPT
- ▶ $W[t]$ for all $t > 0$
- ▶ XNLP
- ▶ XP

XNLP-complete problems

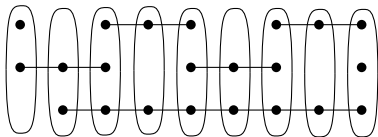
Usually XNLP-complete problems exhibit some form of linear structure. A lot of local choices have to be made, the corresponding local problem is at least $W[1]$ -hard.

Interest of XNLP

Simple definition.

Many complete problems for several standard parameters.

A very important standard linear structure parameter: *pathwidth*

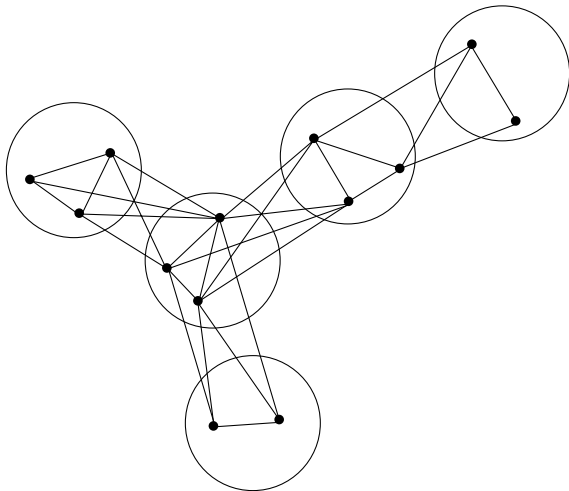


Results

- ▶ Many more XNLP-complete problems
- ▶ XALP: a simple extension of XNLP to tree-like structures

XALP lifts results on pathwidth to results on treewidth “for free”.

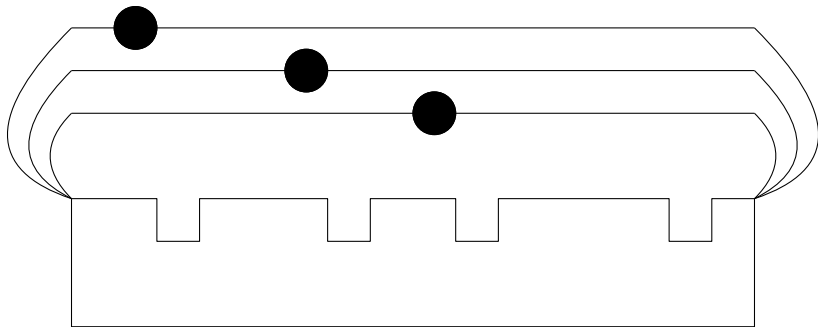
Definition



Hardness

Computing exactly the tree-partition-width is XALP-complete.

Main gadget : “clique-chains”



Approximation

- ▶ First compute a tree decomposition
- ▶ Find highly connected pairs of vertices
- ▶ Compute tree partitions on convenient auxiliary graphs
- ▶ Combine them into a final tree-partition

Results

- ▶ More XNLP-complete problems
- ▶ XALP for the more common tree-like structures
- ▶ Found problems that do not fit into XNLP/XALP
- ▶ Renewed interest for tree-partition-width

Questions

