

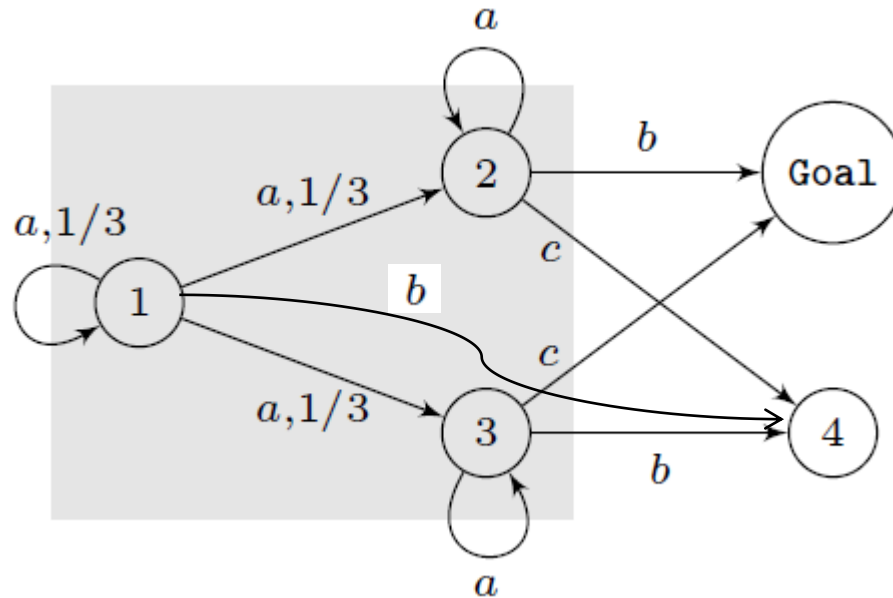
Controlling MDPs vs Family of MDPs vs Probabilistic Automata

Blaise Genest, SUMO

MDP:

Controller chooses action.

Stochastic moves once action is chosen.



Strategy: play *a* long enough till end up in state 2 or 3.

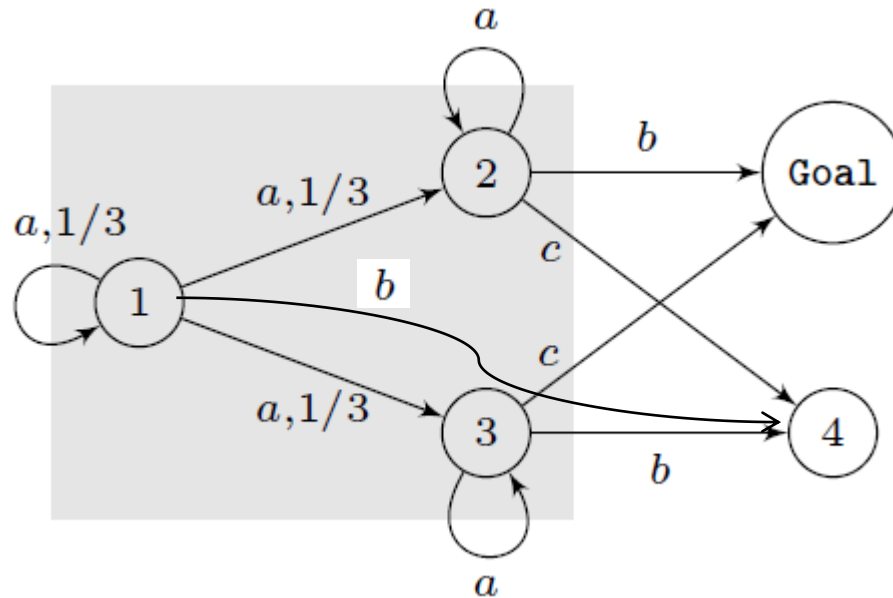
If in 2 play *b*

If in 3 play *c*.

State Goal reached with probability 1.

Probabilistic Automata:

PA = Blind (PO)MDP = MDP where we dont know the state



No Strategy to reach Goal with proba 1.

Dont know if not in 1. Dont know if in 2 or in 3.

Strategy only based on time.

Can compute distribution of proba to be in each state after n steps.

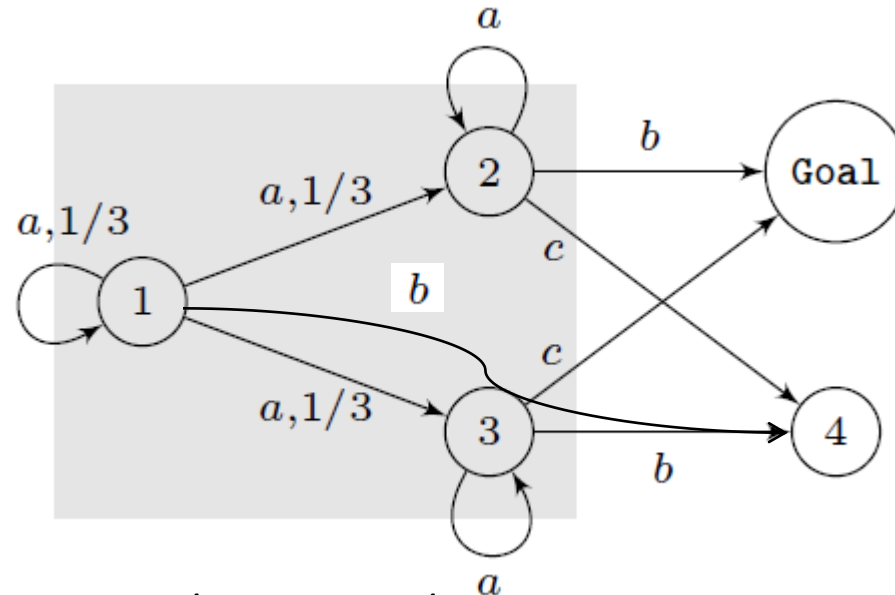
population of MDPs (MDP with Proportions):

Population in state 1.

Proportion of cells in state 1 at time 1 = 1

Playing a:

Proportion of cells in 1 = 1/3, in 2 = 1/3, in 3 = 1/3...



1		1/3		1/9	
0	→	1/3	→	4/9	-----
0		1/3		4/9	

Strategy only based on time.

Can compute distribution of proba to be in each state after n steps.

PA vs Population of MDPs:

Different questions:

PA

Path based logic (reach., PCTL):

Ex: At least 50% of paths
« eventually always Goal».

$\text{Prob}(\text{eventually always Goal}) > 0.5$

Population of MDPs

Distribution based logic:

Ex: Prop States in X1 is always >0.5

Always $\text{Prop}(\text{Goal}) > 0.5$

Orthogonal questions

Decidability for PA?

Undecidable to know whether there exists a strategy ensuring probability 0.5 to reach a state . [Paz'71]

Worse: cannot approximate the probability to reach a state.
[Madani, Hanks, Condon AI 2003]

Decidability only for some qualitative questions
(reachability proba=1,safety >0...).

Approximation of PAs

Undecidable to know whether there exists a strategy ensuring probability 0.5 to reach a state . [Paz'71]

Reason: infinite precision.

Idea: what if we have finite precision?
For all finite precision, can we find a strategy?

Talk of Hugo

Approximation of proportions

Does there exist strategy σ which always satisfies
«Proportion to be in Goal $> \frac{1}{2}$ » (orthogonal to reachability)

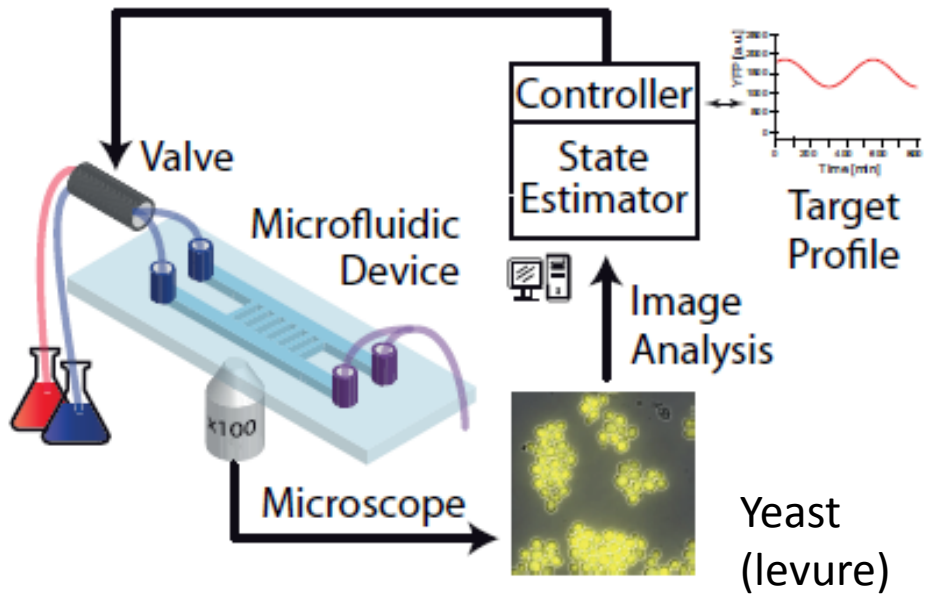
Harder than Skolem

\Rightarrow For all ϵ , does there exist strategy σ_ϵ s.t.
«Proportion to be in $x_1 > \frac{1}{2} + \epsilon$ »

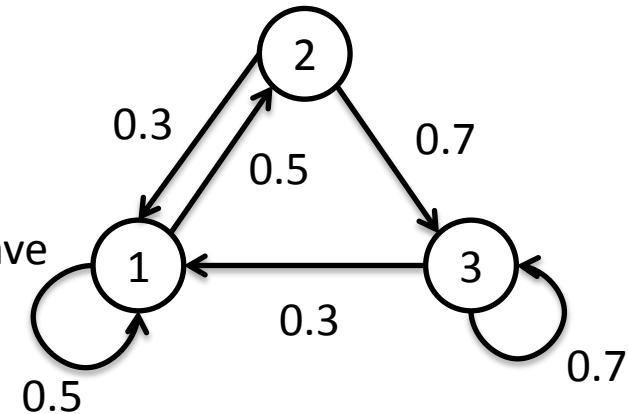
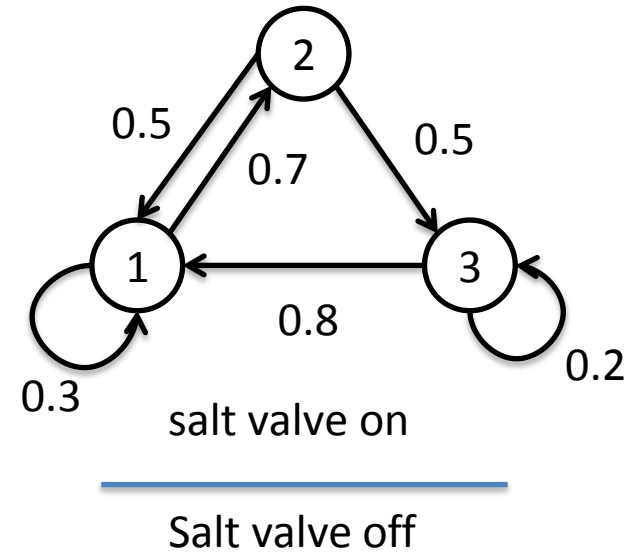
Should be Decidable.

Actually, if strategy fixed,
always «Proportion to be in $x_1 > \frac{1}{2}$ » **Talk of Bruno**
Equivalent to Skolem

Approximation of population of MDPs



Modelization
Parameter
estimation



Setting: Many yeast. Each yeast is in one state 1,2,3.

Question: Based on global fluorescence level, what action to have half of the population with high fluorescence?

Is there a strategy for all finite number (not infinite) of yeasts?

Population of MDPs:

Is there a strategy for all number of yeasts?

=> Give rise to big MDPs

Talks of Ludo and Steve now