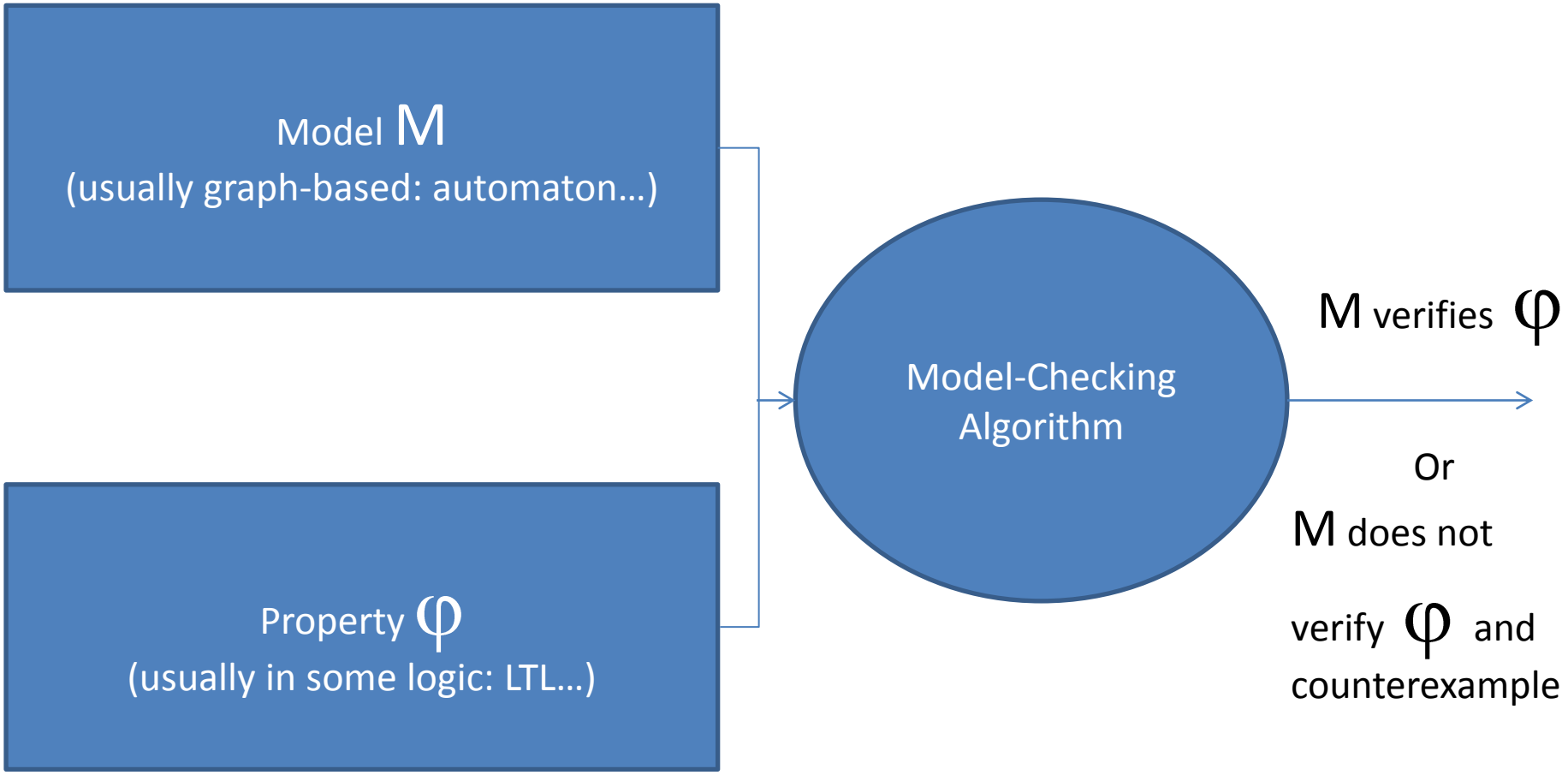
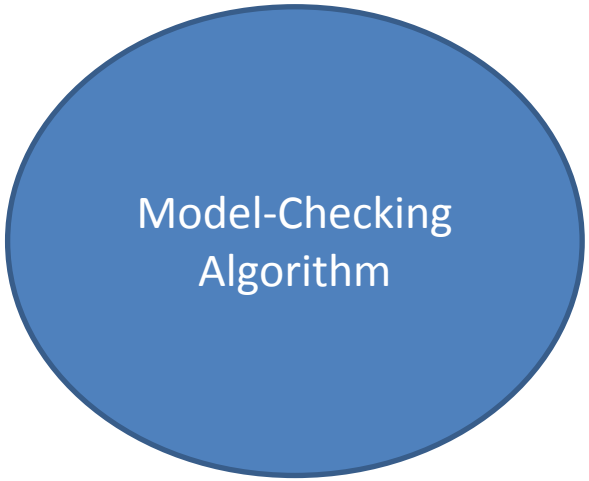
Overview of the ANR project

Model-Checking



[Clarke - Emerson - Siffakis in the 80's]. Turing award 2007

Why Turing Award?



+

BDD

CEGAR

Abstract Interpretation

…

Model Reduction Techniques

Processor

Rocket, Planes → Model Checkers (Blast, NuSMV, SPIN, PRISM…) → Verification, Bugs…

C program code

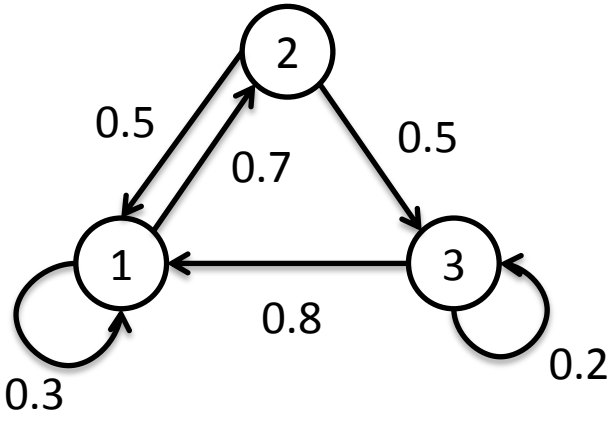
=> Model Reduction for Stochastic Systems

Stochastic Models:

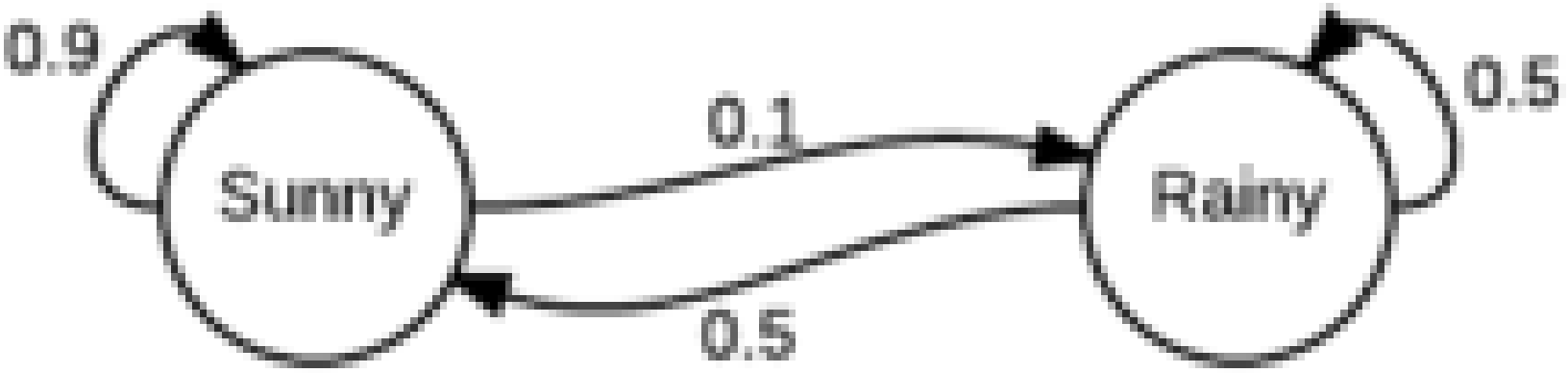
Markov Chains

- It can be represented as a matrix or labeled graph.
  - Nodes represent states.
  - Edges represent influences.
  - Stochastic values represent the uncertainty level.

$$M = \begin{pmatrix} 0.3 & 0.7 & 0 \\ 0.5 & 0 & 0.5 \\ 0.8 & 0 & 0.2 \end{pmatrix}$$



Markov Chains examples



$P(X^{t=10} = \text{Rainy} \mid X^{t=0} = \text{Sunny}) = 0.15\ldots$

Bio-chemical context:

P(Concentration of Insulin is small after 1 hour| concentrations at t=0)?

Semantics and Logics

2 semantics: runs/samples vs distribution

Taking (infinitely) many samples following the transition probability:

Sunny sunny…
Sunny sunny…
Sunny sunny…
Sunny rainy….

Then average the samples.

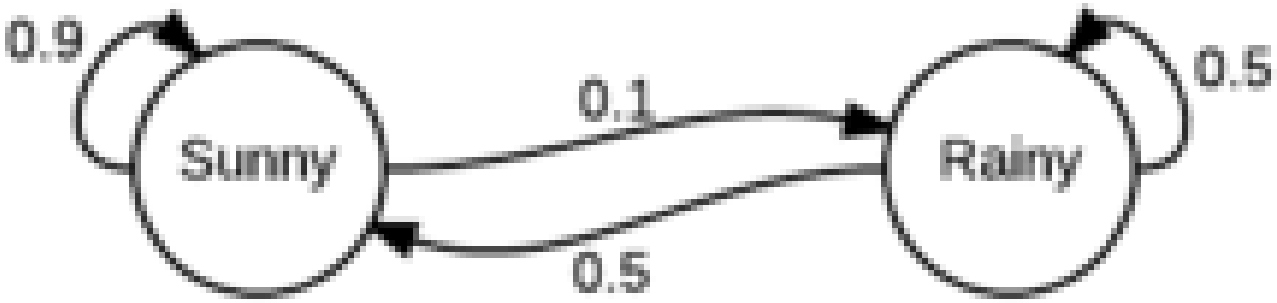
Initial distribution: Start from sunny (1,0)

After 1 step: (0.9,0.1)

After 2 steps: (0.855,0.145)

…

Proportion of samples ending in « sunny » after 2 steps = distribution of sunny after 2 steps



2 kinds of logics:

Path based logic:

Proportion of paths satisfying some constraints

Ex of Paths: sunny sunny…;
sunny rainy…

ex prop: path « eventually always sunny ».

P(eventually always sunny) =0

=> Give rises to logics as PCTL(*)

Distribution based logic:

There is a sequence of probability distributions generated by model satisfying some property

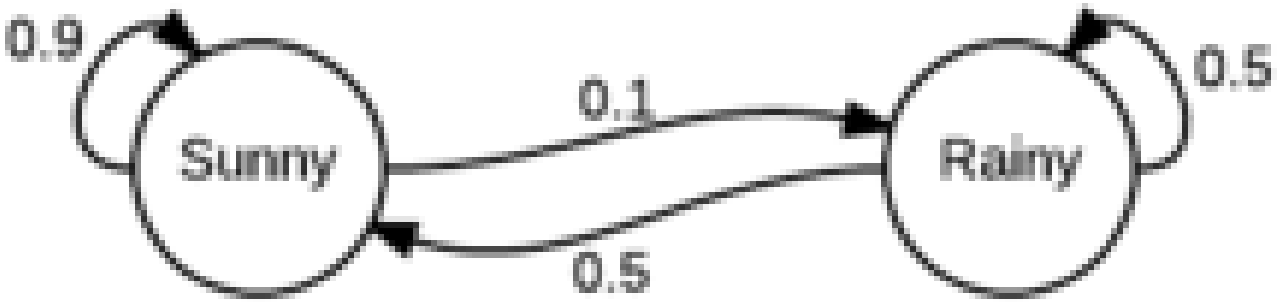
ex of sequence of probability:

(1,0) (0.9,0.1) (0.855,0.145)….

(0.5,0.5) (0.7,0.3) (0.78,0.12)….

Ex of prop:
X_0[sunny]>=0.7 => always X[sunny]>=0.7.

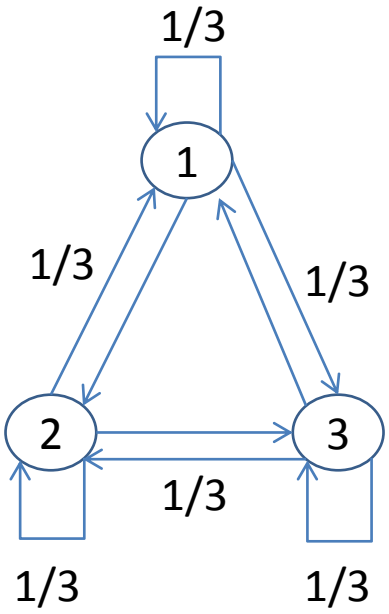
True.



Uncomparable logics:

Distribution based logic vs Path based logic

φ = « next eventually X[1]>0.7» cannot be expressed by PCTL*:



M does not satisfy φ (from any initial distrib)

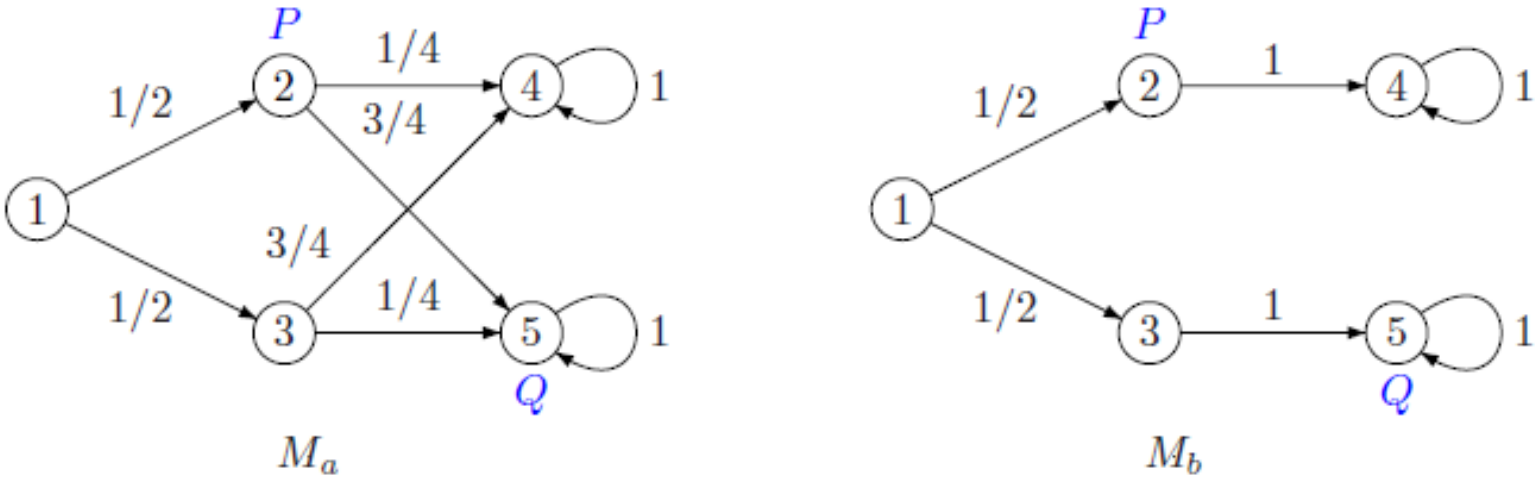
For instance, P(next eventually 1) > 0.7 is true for M (from any state)

[Beauquier Rabinovitch Slissenko CSL'02]

Uncomparable logics:

Path based logics vs Distribution based logics:

Some PCTL formula cannot be expressed by distribution based logics



Same sequence of distribution for both chain from distribution X[1]=1.
But can be distinguished by formula of PCTL: Prob (Next P Until Q)>1/9.

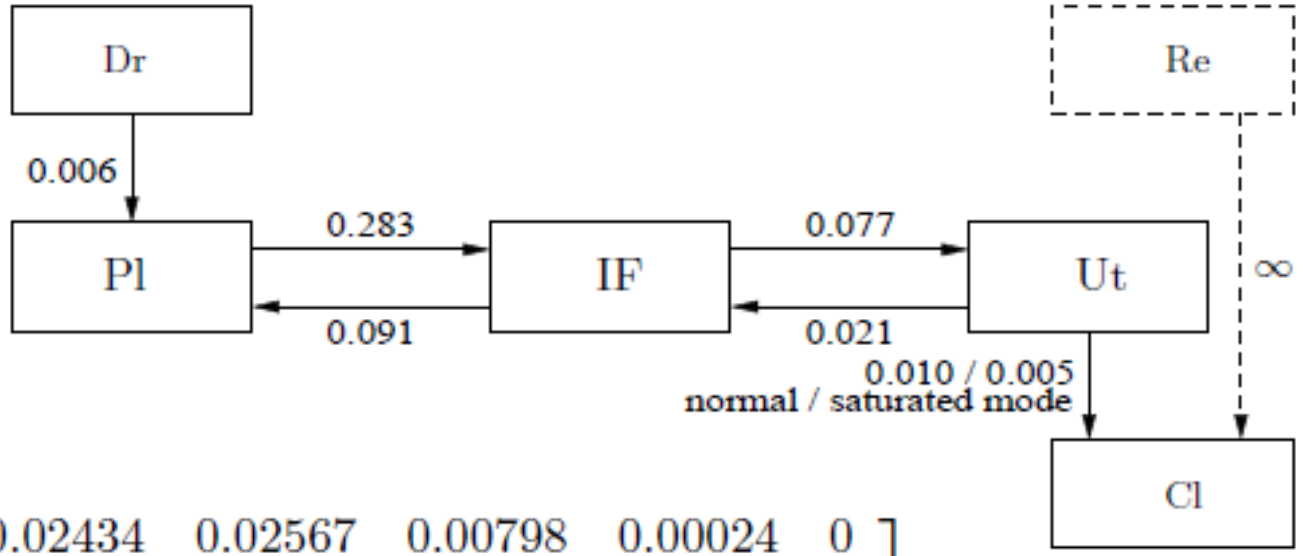
[Beauquier Rabinovitch Slissenko CSL'02]

Biochemical Context:

P(Concentration of Insulin is small after 1 hour| concentrations at t=0)

Insulin:

Body=3 parts:



$$N = \begin{bmatrix} 0.94 & 0.02434 & 0.02567 & 0.00798 & 0.00024 & 0 \\ 0.00 & 0.20724 & 0.48298 & 0.29624 & 0.01353 & 0 \\ 0.00 & 0.15531 & 0.42549 & 0.39520 & 0.02400 & 0 \\ 0.00 & 0.02598 & 0.10778 & 0.77854 & 0.08770 & 0 \\ 0.00 & 0.00000 & 0.00000 & 0.00000 & 1.00000 & 0 \\ 0.00 & 0.00000 & 0.00000 & 0.00000 & 1.00000 & 0 \end{bmatrix}$$

Distrib. Based logic

Model Checking of Stochastic Systems

Results on PCTL(*):

PCTL decidable in polynomial time for Markov Chains (even for MDPs)
[*Hansson and Jonsson 94 + Bianco and de Alfaro 95*]

PCTL^* decidable in exponential time for MC (doubly exp time for MDPs)

$\Rightarrow$ 1) up to 10^10 states (e.g. [PRISM])
$\Rightarrow$ 2) not for distribution based logics

Results on distribution based logics:

[Ouaknine-Worrel'14]:
«Eventually X[i]>=p » is decidable for Markov Chains with 6 states.
(comes from decidability of positivity of the skolem problem).
In case all eigen values of M are distinct, decidable for 10 states.

«Eventually always X[i] >=p» is decidable for Markov Chains with 6 states
(comes from decidability of ultimate positivity of the skolem problem).
In case all eigen values of M are distinct,
decidable for all Markov Chains.

Decidability for 16 states would entail major breakthrough about diophantine approximation of transcendental numbers

General Case: as hard as Skolem problem (open for 40 years).

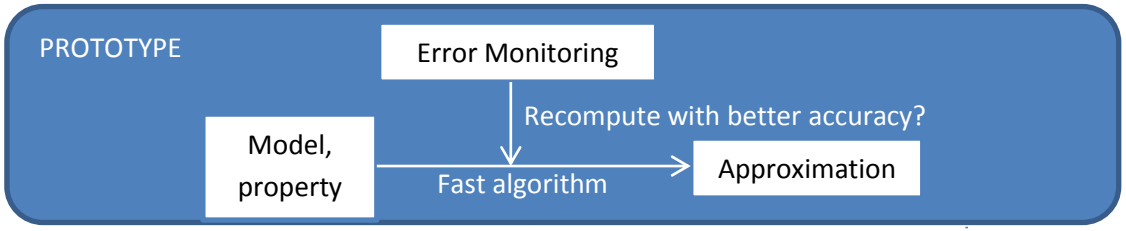
Model Checking for Stochastic Systems based on approximate algorithms

Model Checking de systèmes stochastiques

Approximate the values.

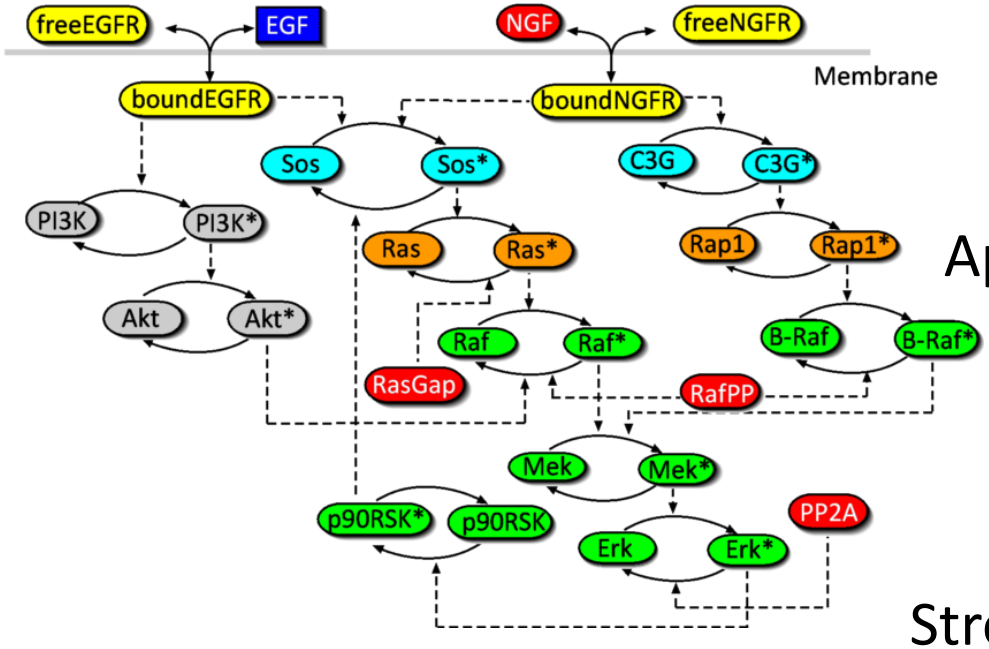
Instead of e.g. Probability = 0.3, we answer e.g. Proba in [0.2,0.4].

If question was: is probability < 0.5, then sufficient.



Model Checking de systèmes stochastiques

Computational System Biology (biopathways)



Applications to:

Apoptosis in Hela cells (large)

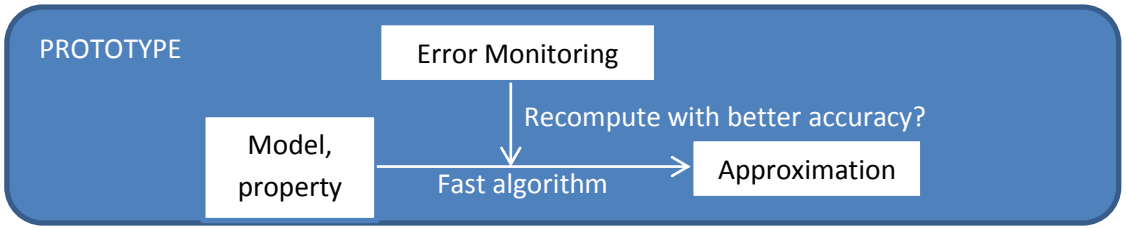
Stress response in yeast (choices)

PROGRAMME BLANC/JCJC/BLIN 2013

General Idea

Approximate the values.

Instead of e.g. Probability = 0.3, we answer e.g. Proba in [0.2,0.4].
If question was: is probability < 0.5, then sufficient.



Multiple frameworks:

- Bounded Model Checking of big Dynamic Bayesian Networks
- Model Checking of Markov Chains vs distribution based logics
- Model Checking of MDP vs distributions ? [Chada et al.'11]

Concluding Remarks

Conclusion

Approximation => pragmatic exact model checking for stochastic models

- large DBN models (biological pathways with 10^20 states).

[Palaniappan, Akshay, Liu, G., Thiagarajan TCBB 2012]

- Model Checking of distribution based prop for MC (Skolem problem).
- Even MDPs.

Know Limits: cannot prove value 1/ probability 1 (but other techniques can)

[Chatterjee et al.]

Different from PCTL^* model checking but can reuse some techniques.

Lots of Mathematical results but:

Still a lot to do. Not many people looking at the problem from formal methods community. Mostly statistical methods, similar but not equivalent to ours.