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(7)

$$\begin{array}{r|l} 2x^2 - 6x + 5 & x - 1 \\ - (2x^2 - 2x) & 2x - 4 \\ \hline -4x + 5 & \\ - (-4x + 4) & \\ \hline 1 & \end{array}$$

$$2x^2 - 6x + 5 = (x-1)(2x-4) + 1$$

~~$$\begin{array}{r} 2x^2 - 6x + 5 \\ \hline 2x - 4 \\ \hline x + 1 \end{array}$$~~

$$\left(\frac{2x^2 - 6x + 5}{x-1} = 2x - 4 + \frac{1}{x-1} \right)$$

$$\textcircled{1} \quad a = \frac{1}{2 + \frac{1}{1 + \frac{1}{3+a}}}$$

$$\frac{3+a}{3+a} + \frac{1}{3+a} = \frac{4+a}{3+a}$$

$$a = \frac{1}{2 + \frac{1}{\frac{4+a}{3+a}}} = \frac{1}{2 + \frac{3+a}{4+a}} = \frac{1}{\frac{8+2a+3+a}{4+a}} = \frac{4+a}{11+3a}$$

$$a = \frac{4+a}{11+3a} \Leftrightarrow a(11+3a) = 4+a \Leftrightarrow 3a^2 + 11a = 4+a$$

$$\Leftrightarrow 3a^2 + 10a - 4 = 0$$

$$\Delta = \dots$$

$$\textcircled{6} 9 + \dots + 37 = \sum_{k=3}^{18} (k+1) = 2 \left(\sum_{k=3}^{18} k + \sum_{k=3}^{18} 1 \right)$$

nombre de termes

$$= 2 \cdot \frac{(1^{\text{er}} + \text{dernier}) \cdot (\text{nb de termes})}{2} + 18 - 3 + 1 = \dots$$

⑪ $\binom{n}{p} = \frac{n!}{p!(n-p)!}$ Dans le triangle de Pascal, on additionne des entiers, donc c'est un entier

$$(2 + \sqrt{5})^n = \binom{n}{0} 2^n + \binom{n}{1} 2^{n-1} \sqrt{5} + \binom{n}{2} 2^{n-2} (\sqrt{5})^2 + \dots + \binom{n}{n} (\sqrt{5})^n$$

$$(2 - \sqrt{5})^n = \binom{n}{0} 2^n - \binom{n}{1} 2^{n-1} \sqrt{5} + \binom{n}{2} 2^{n-2} (\sqrt{5})^2 - \dots$$

$$+ = 2 \binom{n}{0} 2^n + 0 + 2 \binom{n}{2} 2^{n-2} (\sqrt{5})^2 + 0 + \dots$$

$$\textcircled{2} \quad P(x) = \underbrace{x^2 - 49}_{a^2 - b^2 = (a+b)(a-b)} + (3x-4)(-x-7) + 2(2x+14)$$

$$\begin{aligned} P(x) &= \underbrace{(x+7)}_{\swarrow} \underbrace{(x-7)}_{\searrow} - (3x-4)\underbrace{(x+7)}_{\swarrow} + 4\underbrace{(x+7)}_{\swarrow} \\ &= (x+7) [x-7 - (3x-4) + 4] \end{aligned}$$

$$-x-7 = -(x+7)$$

$$(5) \sum_{k=0}^n \binom{j-n}{j-k} 2^k = \sum_{k=0}^n (2^k + 2^{k+1} + \dots + 2^n)$$

$$1 + x + \dots + x^n = \frac{x^{n+1} - 1}{x - 1} \quad \text{for } x \neq 1$$

~~$$\frac{x^{n+1} - 1}{x - 1}$$~~

$$2^k (1 + 2 + 2^2 + \dots + 2^{n-k}) = 2^k \left(\frac{2^{n-k+1} - 1}{2 - 1} \right)$$

Equa Diff d'ordre 1

1) Rappel de calcul intégral

1.1. Primitives usuelles

$$\int x^n dx = \frac{x^{n+1}}{n+1} + \text{cte} \text{ pour } n \neq -1$$

$$\int \frac{1}{x} dx = \text{Ln} |x| + \text{cte}$$

$$\int e^x dx = e^x + \text{cte}$$

$$\int \cos x dx = \sin x + \text{cte}$$

$$\int \sin x dx = -\cos x + \text{cte}$$

$$(x^a)' = ax^{a-1}$$

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$$(x^{n+1})' = (n+1)x^n$$

$$\Rightarrow \int \frac{u'}{u} = \text{Ln} |u| + \text{cte}$$

$$\int e^{ax} dx = \frac{e^{ax}}{a} + \text{cte}$$

$$\int \cos(ax) dx = \frac{\sin(ax)}{a} + \text{cte}$$

12. Intégration par partie (I&I)

$$\int u'v = [uv] - \int uv'$$

ex 1: $\int_2^{37} \arctan(t) dt = I ?$ $\arctan(t)' = \frac{1}{1+t^2}$

$I = \int_2^{37} 1 \cdot \arctan(t) dt$ I&I $\begin{cases} u' = 1 & u = t \\ v = \arctan t & v' = \frac{1}{1+t^2} \end{cases}$

$$I = [t \cdot \arctan(t)]_2^{37} - \int_2^{37} t \cdot \frac{1}{1+t^2} dt$$
$$= 37 \arctan(37) - 2 \arctan 2 - \int_2^{37} \frac{t}{1+t^2} dt$$

$$I = 37 \operatorname{arctan}(37) - 2 \operatorname{arctan}(2) - \frac{1}{2} \int_2^{37} \frac{2t}{1+t^2} dt$$

$$= \quad " \quad " \quad - \frac{1}{2} \left[\ln(1+t^2) \right]_2^{37}$$

$$= \quad " \quad " \quad - \frac{1}{2} (\ln(1+37^2) - \ln(1+2^2))$$

ex 2: $I = \int x \sin(2x) dx$ I & R R $\left\{ \begin{array}{l} u = x \quad u' = 1 \\ v' = \sin(2x) \quad v = -\frac{\cos(2x)}{2} \end{array} \right.$

$$I = \left[-\frac{x \cos(2x)}{2} \right] - \int 1 \cdot \frac{-\cos(2x)}{2} dx = -\frac{x \cos(2x)}{2} + \frac{1}{2} \int \cos(2x) dx$$

$$I = -x \frac{\cos(2x)}{2} + \frac{1}{2} \frac{\sin(2x)}{2} + \text{cte} = \frac{\sin(2x) - 2x \cos(2x)}{4} + \text{cte}$$

$$\text{ex 3 } C = \int \cos x e^x dx$$

$$\text{I P P} \left| \begin{array}{l} u' = \cos x \\ v = e^x \end{array} \right. \quad \begin{array}{l} u = \sin x \\ v' = e^x \end{array}$$

$$C = \underbrace{[\sin x e^x]} - \underbrace{\int \sin x e^x dx}$$

$$\text{I P P} \left| \begin{array}{l} u' = \sin x \\ v = e^x \end{array} \right. \quad \begin{array}{l} u = -\cos x \\ v' = e^x \end{array}$$

$$C = \sin x e^x - \left([\cos x e^x] - \int -\cos x e^x dx \right)$$

$$C = \sin x e^x + \cos x e^x - \int \cos x e^x dx$$

$$C = \sin x e^x + \cos x e^x - C$$

$$\Rightarrow 2C = \sin x e^x + \cos x e^x \Rightarrow$$

$$C = \frac{\sin x e^x + \cos x e^x}{2}$$

$$C = \frac{(\sin x + \cos x) e^x}{2}$$

1.3 Changement de variable

$$\text{ex 1: } I = \int \frac{2t}{1+t^2} dt$$

$$\text{changement : } u = 1+t^2$$

$$u' = \frac{du}{dt} = 2t \Rightarrow du = 2t dt$$

$$\Rightarrow \boxed{dt = \frac{du}{2t}}$$

$$I = \int \frac{\cancel{2t}}{u} \frac{du}{\cancel{2t}} = \int \frac{1}{u} du = \text{Ln}|u| + \text{cte} = \text{Ln}(1+t^2) + \text{cte}$$

$$\text{ex 2: } I = \int_0^{\sqrt{\pi/2}} t \cos(t^2) dt$$

$$\text{chgt } \boxed{u = t^2} \quad \frac{du}{dt} = 2t$$

$$\triangle \text{ les bornes changent : qd } t=0, u=0 \text{ et qd } t=\sqrt{\pi/2}, u=\pi/2 \quad du = 2t dt \Rightarrow \boxed{dt = \frac{du}{2t}}$$

$$I = \int_0^{\pi/2} \cancel{t} \cos(u) \frac{du}{\cancel{2t}} = \frac{1}{2} \int_0^{\pi/2} \cos(u) du = \frac{1}{2} [\sin(u)]_0^{\pi/2}$$

$$I = \frac{1}{2} (\sin(\frac{\pi}{2}) - \sin(0)) = \frac{1}{2} (1-0) = \frac{1}{2}$$

ex 3: $k = \int_{t=2}^{t=3} \frac{1}{t \ln(t)} dt$

chgt $u = \ln(t)$

$$\frac{du}{dt} = \frac{1}{t} \Rightarrow dt = t du$$

$$k = \int_{u=\ln(2)}^{u=\ln(3)} \frac{1}{u} du = \left[\ln(u) \right]_{\ln(2)}^{\ln(3)} = \underline{\ln(\ln(3)) - \ln(\ln(2))}$$

qd $t=2$ $u = \ln(2)$

II Equations Différentielles Linéaires

2.1 L'équation homogène

On considère \tilde{a} : $a(x) y'(x) + b(x) y(x) = 0$

sol: $y_H = A e^{-\int \frac{b}{a}}$