

well

$$(1) \begin{cases} u_{n+1} = -3u_n + v_n + 3w_n \\ v_{n+1} = -4u_n + v_n + 4w_n \\ w_{n+1} = -2u_n + v_n + 2w_n \end{cases}$$

$$\underbrace{\begin{pmatrix} -3 & 1 & 3 \\ -4 & 1 & 4 \\ -2 & 1 & 2 \end{pmatrix}}_A \begin{pmatrix} u_n \\ v_n \\ w_n \end{pmatrix} = \begin{pmatrix} -3u_n + v_n + 3w_n \\ \text{---} \\ \text{---} \end{pmatrix} = \begin{pmatrix} u_{n+1} \\ v_{n+1} \\ w_{n+1} \end{pmatrix}$$

(2) $x_{n+1} = Ax_n$, donc $(x_n)_{n \in \mathbb{N}}$ est une suite géométrique.

donc $x_n = A^n x_0$

(3) J'appelle $\mathcal{B} = (\vec{i}, \vec{j}, \vec{k})$ la base canonique de \mathbb{R}^3
J'appelle f l'endomorphisme de \mathbb{R}^3 tel que $\text{Mat}_{\mathcal{B}}(f) = A$

$$f(E_1) = A \times E_1 = \begin{pmatrix} -3 & 1 & 3 \\ -4 & 1 & 4 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} = - \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = -1E_1$$

$f(E_1) = -1E_1$

donc E_1 est un vecteur propre pour la valeur propre -1

$$\textcircled{4a} \quad \Delta_A(x) = \begin{vmatrix} -3-x & 1 & 3 \\ -4 & 1-x & 4 \\ -2 & 1 & 2-x \end{vmatrix} = \begin{vmatrix} -x & 1 & 3 \\ 0 & 1-x & 4 \\ -x & 1 & 2-x \end{vmatrix} \xrightarrow[\substack{C_1+C_3 \\ L_3-L_1}]{L_3-L_1} \begin{vmatrix} -x & 1 & 3 \\ 0 & 1-x & 4 \\ 0 & 0 & -1-x \end{vmatrix}$$

$$\stackrel{\text{det}}{=}_{C_1} -x \begin{vmatrix} 1-x & 4 \\ 0 & -1-x \end{vmatrix} = -x(1-x)(-1-x)$$

3 valeurs prop: $-1, 0$ et 1

(4c) Espaço nulo para 0: $\left\{ \vec{u} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 / f(\vec{u}) = 0 \vec{u} \right\} = \ker(B)$

$$f(\vec{u}) = \vec{0} \Leftrightarrow \begin{cases} -3x + y + 3z = 0 \\ -4x + y + 4z = 0 \\ -2x + y + 2z = 0 \end{cases} \xrightarrow{L_1 - L_2} \begin{cases} -3x + y + 3z = 0 \\ x - z = 0 \\ -2x + y + 2z = 0 \end{cases} \xrightarrow{L_1 - L_3} \begin{cases} -3x + y + 3z = 0 \\ x - z = 0 \\ -x + y = 0 \end{cases} \xrightarrow{\text{dein 1}} \begin{cases} -3x + y + 3z = 0 \\ -x + y = 0 \end{cases}$$

Sei $x=1$ (é livre) $\Rightarrow z=1 \Rightarrow y=0$

(4d) $f(\vec{u}) = 1 \vec{u} \Leftrightarrow \begin{cases} -3x + y + 3z = x \\ -4x + y + 4z = y \\ -2x + y + 2z = z \end{cases} \Leftrightarrow \begin{cases} -4x + y + 3z = 0 \\ -4x + y + 4z = 0 \\ -2x + y + z = 0 \end{cases}$

$$E_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} -4x + y + 3z = 0 \\ -x + z = 0 \end{cases} \xrightarrow{L_1 - L_2} \begin{cases} -4x + y + 3z = 0 \\ -x + z = 0 \end{cases}$$

$x=1 \Rightarrow z=1 \Rightarrow y=1$

$$E_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

(5) $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$P = \begin{pmatrix} E_1 & E_2 & E_3 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

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$$f(x) = |\cos x| - |\sin x|$$

$$\cos(x+\pi) = -\cos x \quad \text{et} \quad \sin(x+\pi) = -\sin x$$

$$\textcircled{1} f(x+\pi) = |-\cos x| - |-\sin x| = |\cos x| - |\sin x| = f(x). \quad \text{Période } \pi$$

$$f(-x) = |\cos(-x)| - |\sin(-x)| = |\cos x| - |-\sin x| = |\cos x| - |\sin x| = f(x) \quad \text{fonction paire}$$

$$\cos(-x) = \cos x \quad \text{et} \quad \sin(-x) = -\sin x$$

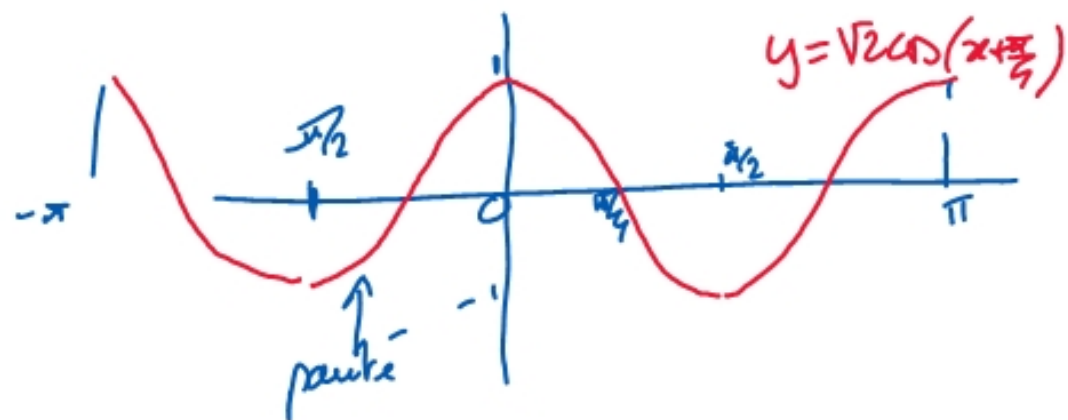
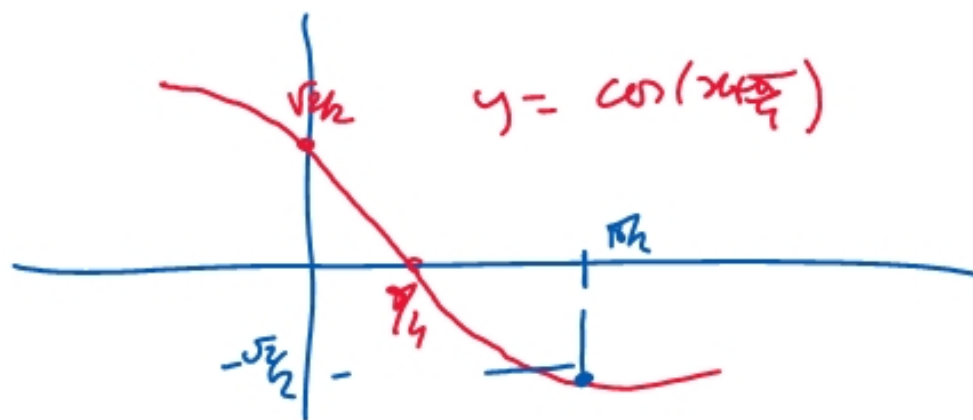
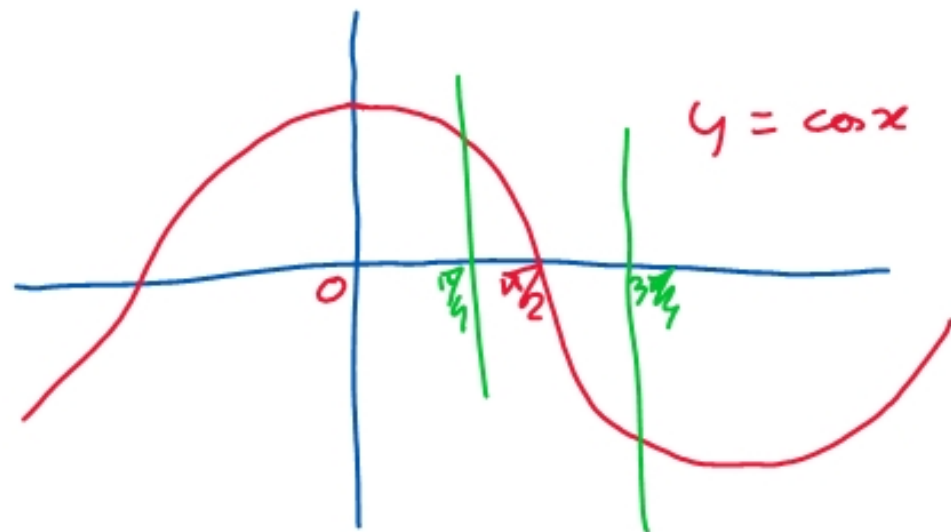
$$\textcircled{2} \begin{array}{ll} |\cos x| = \cos x & \text{sur } [0, \frac{\pi}{2}] \\ |\sin x| = \sin x & \text{sur } [0, \frac{\pi}{2}] \end{array} \quad \begin{array}{ll} |\cos x| = -\cos x & \text{sur } [\frac{\pi}{2}, \pi] \\ |\sin x| = \sin x & \text{sur } [\frac{\pi}{2}, \pi] \end{array}$$

$$\text{donc } |\cos x| - |\sin x| = \cos x - \sin x \quad \forall x \in [0, \frac{\pi}{2}]$$

$$\textcircled{3} C \cos(x + \frac{\pi}{4}) = C \left[\cos(x) \cos(\frac{\pi}{4}) - \sin(x) \sin(\frac{\pi}{4}) \right] = C \frac{\sqrt{2}}{2} (\cos x - \sin x)$$

$$\Rightarrow C = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$f(x) = \sqrt{2} \cos(x + \frac{\pi}{4}) \quad \forall x \in [0, \frac{\pi}{2}]$$



$$① \begin{cases} U_{n+1} = -3U_n + V_n + 3W_n \\ V_{n+1} = -4U_n + V_n + 4W_n \\ W_{n+1} = -2U_n + V_n + 2W_n \end{cases} \leftarrow$$

$$\underbrace{\begin{pmatrix} -3 & 1 & 3 \\ -4 & 1 & 4 \\ -2 & 1 & 2 \end{pmatrix}}_A \begin{pmatrix} U_n \\ V_n \\ W_n \end{pmatrix} = \begin{pmatrix} -3U_n + V_n + 3W_n \\ -4U_n + V_n + 4W_n \\ -2U_n + V_n + 2W_n \end{pmatrix} = \begin{pmatrix} U_{n+1} \\ V_{n+1} \\ W_{n+1} \end{pmatrix}$$

② $X_{n+1} = AX_n$, donc $(X_n)_{n \in \mathbb{N}}$ est une suite géométrique, de raison A

donc $X_n = A^n X_0$

③ J'appelle $\mathcal{B} = (\vec{i}, \vec{j}, \vec{k})$ la base canonique de \mathbb{R}^3
 J'appelle f l'endomorphisme de \mathbb{R}^3 tel que $\text{Mat}_{\mathcal{B}}(f) = A$

$$f(E_1) = A \times E_1 = \begin{pmatrix} -3 & 1 & 3 \\ -4 & 1 & 4 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} = - \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = -1E_1$$

donc E_1 est un vecteur propre pour la valeur propre $d_1 = -1$

$$\textcircled{4a} \quad P_A(X) = \begin{vmatrix} -3-X & 1 & 3 \\ -4 & 1-X & 4 \\ -2 & 1 & 2-X \end{vmatrix} \underset{C_1+C_3}{=} \begin{vmatrix} -X & 1 & 3 \\ 0 & 1-X & 4 \\ -X & 1 & 2-X \end{vmatrix} \underset{L_3-L_1}{=} \begin{vmatrix} -X & 1 & 3 \\ 0 & 1-X & 4 \\ 0 & 0 & -1-X \end{vmatrix}$$

$$\underset{C_1}{\stackrel{\text{d'apr}}{=}} \quad -X \begin{vmatrix} 1-X & 4 \\ 0 & -1-X \end{vmatrix} = -X(1-X)(-1-X)$$

3 valeurs propres : $-1, 0, \text{ et } 1$

(4) Espace propre pour 0. $E_0 = \left\{ \vec{u} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 / f(\vec{u}) = 0 \cdot \vec{u} \right\} = \text{Ker}(f)$

$$f(\vec{u}) = \vec{0} \Leftrightarrow \begin{cases} -3x + y + 3z = 0 \\ -4x + y + 4z = 0 \\ -2x + y + 2z = 0 \end{cases} \begin{matrix} \Leftrightarrow 4 - L_2 \\ \Leftrightarrow 4 - L_3 \end{matrix} \begin{cases} -3x + y + 3z = 0 \\ x - z = 0 \\ -x + z = 0 \end{cases} \begin{matrix} \Leftrightarrow -z = 0 \\ +z = 0 \end{matrix} \begin{matrix} \Leftrightarrow \\ \text{d'où } 1 \end{matrix} \begin{cases} -3x + y + 3z = 0 \\ -x + z = 0 \end{cases}$$

$\forall \epsilon_i \quad x=1 \text{ (choix)} \Rightarrow z=1 \Rightarrow y=0$

$$E_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

(4d) $f(\vec{u}) = 1 \cdot \vec{u} \Leftrightarrow \begin{cases} -3x + y + 3z = x \\ -4x + y + 4z = y \\ -2x + y + 2z = z \end{cases} \Leftrightarrow \begin{cases} -4x + y + 3z = 0 \\ -4x + y + 4z = 0 \\ -2x + y + z = 0 \end{cases}$

$$\Leftrightarrow \begin{cases} -4x + y + 3z = 0 \\ -4x + y + 4z = 0 \\ -2x + y + z = 0 \end{cases} \begin{matrix} \Leftrightarrow \\ 4 - L_3 \end{matrix} \begin{cases} -4x + y + 3z = 0 \\ -4x + y + 4z = 0 \\ -2x + y + z = 0 \end{cases} \Leftrightarrow \begin{cases} -4x + y + 3z = 0 \\ -x + z = 0 \end{cases}$$

$x=1 \Rightarrow z=1 \Rightarrow y=1$

$$E_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

(5) $D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$P = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$
 $\begin{matrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{matrix}$

$$\textcircled{6} \quad A = P D P^{-1}$$

$$A^m = (P D P^{-1})^m = \overbrace{P D P^{-1} \cdot P D P^{-1} \cdot P D P^{-1} \cdots P D P^{-1}}^{m \text{ fois}}$$

$$= P \cdot \underbrace{D \cdot D \cdots D}_{D^m} \cdot P^{-1} = P D^m P^{-1}$$

$$D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$D^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$D^4 = D^2$$

$$D^3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = D$$

$$D^{2k} = D^2$$

$$\text{et } D^{2k-1} = D$$

$$A^m = P D^m P^{-1} \begin{cases} = P D P^{-1} = A & \text{qd } m \text{ est impair} \\ = P D^2 P^{-1} = A^2 & \text{qd } m \text{ est pair} \end{cases}$$

$$A^2 =$$

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$$\cos(x+\pi) = -\cos x \text{ et } \sin(x+\pi) = -\sin x$$

$$\textcircled{1} f(x+\pi) = |-\cos x| - |-\sin x| = |\cos x| - |\sin x| = f(x). \quad \underline{\text{Période } \pi}$$

$$f(-x) = |\cos(-x)| - |\sin(-x)| = |\cos x| - |-\sin x| = |\cos x| - |\sin x| = f(x) \quad \underline{\text{fonction paire}}$$

$$\cos(-x) = \cos x \text{ et } \sin(-x) = -\sin x$$

$$\textcircled{2} |\cos x| = \cos x \text{ sur } [0, \frac{\pi}{2}] \text{ et } |\cos x| = -\cos x \text{ sur } [\frac{\pi}{2}, \pi]$$

$$|\sin x| = \sin x \text{ sur } [0, \pi]$$

$$\text{donc } |\cos x| + |\sin x| = \cos x - \sin x \quad \forall x \in [0, \frac{\pi}{2}]$$

$$\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b \quad \uparrow_a$$

$$\textcircled{3} C \cos(x + \frac{\pi}{4}) = C \left[\cos(x) \cos(\frac{\pi}{4}) - \sin(x) \sin(\frac{\pi}{4}) \right] = C \frac{\sqrt{2}}{2} (\cos x - \sin x)$$

$$\Rightarrow C = \sqrt{2}$$

$$\underline{f(x) = \sqrt{2} \cos(x + \frac{\pi}{4}) \quad \forall x \in [0, \frac{\pi}{2}]}$$

