

Combinatoire

① $\binom{n}{p} = \frac{n!}{p!(n-p)!}$ = la partie de factoriel

② Somme partielle géométrique

$$1 + x + x^2 + \dots + x^n = \begin{cases} \frac{x^{n+1} - 1}{x - 1} & \text{pour } x \neq 1 \\ n+1 & \text{pour } x = 1 \end{cases}$$

③ Formule permettant d'avancer dans le triangle de Pascal

$$\binom{n}{p} + \binom{n}{p+1} = \binom{n+1}{p+1}$$

④ $\binom{10}{2} = ?$

$$\binom{10}{2} = \frac{10!}{2!8!} = \frac{2 \cdot 3 \cdot \dots \cdot 4 \cdot 8 \cdot 9 \cdot 10}{2 \cdot 2 \cdot 3 \cdot \dots \cdot 8} = \frac{9 \cdot 10}{2} = 45$$

⑤ Formules du binôme de Newton

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

(1)

$$\frac{124}{256} = ?$$

$$\begin{array}{r|l} 124 & 2 \\ 62 & 2 \\ \hline 31 & \end{array}$$

$$124 = 2^2 \cdot 31$$

$$\begin{array}{r|l} 256 & 2 \\ 128 & 2 \\ 64 & 2 \\ 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \end{array}$$

$$256 = 2^8$$

$$\frac{124}{256} = \frac{2^2 \cdot 31}{2^8} = \frac{31}{2^6}$$

$$\frac{35}{63} = \frac{5 \cdot 7}{9 \cdot 7} = \frac{5}{9}$$

(2) $\frac{1}{2} - \frac{5}{6} = \frac{3-5}{6} = \frac{-2}{6} = -\frac{1}{3}$

$$\textcircled{3} \quad \frac{25}{16} \times \frac{4}{5} \times 3 = \frac{5^2 \cdot 2^2 \cdot 3}{2^4 \cdot 5} = \frac{5 \cdot 3}{2^2} = \frac{15}{4}$$

$$\textcircled{4} \quad B = \frac{\frac{4}{5}}{\frac{5}{3}} = \frac{4}{5} \times \frac{3}{5} = \frac{4}{3}$$

$$C = \frac{-5}{\frac{2}{3}} = -5 \times \frac{3}{2} = -\frac{15}{2}$$

$$D = \frac{-\frac{5}{2}}{3} = -\frac{5}{2} \cdot \frac{1}{3} = -\frac{5}{6}$$

$$\textcircled{f} \quad \frac{\frac{2}{3} + \frac{1}{7}}{3 - \frac{1}{3}} = \frac{\frac{14+3}{21}}{\frac{9-1}{3}} = \frac{\frac{17}{21}}{\frac{8}{3}} = \frac{17}{21} \times \frac{3}{8} = \frac{17 \times 3}{3 \times 7 \times 8} = \frac{17}{56}$$

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}} = 1 + \frac{1}{1 + \frac{1}{\frac{3}{2}}} = 1 + \frac{1}{1 + \frac{2}{3}} = 1 + \frac{1}{\frac{5}{3}} = 1 + \frac{3}{5} = \frac{5+3}{5} = \frac{8}{5}$$

$$\frac{\frac{1}{a} + \frac{1}{b}}{1 + \frac{a}{b}} = \frac{\frac{b+a}{ab}}{\frac{b+a}{b}} = \frac{b+a}{ab} \times \frac{b}{b+a} = \frac{1}{a}$$

$$\textcircled{6} \quad a = \frac{1}{3 + \frac{1}{2 + \frac{1}{1+a}}} = \frac{1}{3 + \frac{1}{\frac{2+2a+1}{1+a}}} = \frac{1}{3 + \frac{1+a}{2a+3}}$$

$$a = \frac{1}{\frac{6a+9+1+a}{2a+3}} = \frac{2a+3}{7a+10} \Rightarrow a(7a+10) = 2a+3$$

$$\Rightarrow 7a^2 + 10a = 2a + 3 \Rightarrow 7a^2 + 8a - 3 = 0$$

$$\Delta = 64 + 84 = 148 = 2 \times 74 = 2^2 \times 37 \quad \sqrt{\Delta} = 2\sqrt{37}$$

$$a = \frac{-8 \pm 2\sqrt{37}}{14} = \frac{2(-4 \pm \sqrt{37})}{2 \cdot 7}$$

Formulas de trigo:

$$\cos(-x) = \cos x$$

$$\sin(-x) = -\sin x$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$

$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$

$$\cos(2x) = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\sin(2x) = 2\sin x \cos x$$

$$\sin^2 x + \cos^2 x = 1$$

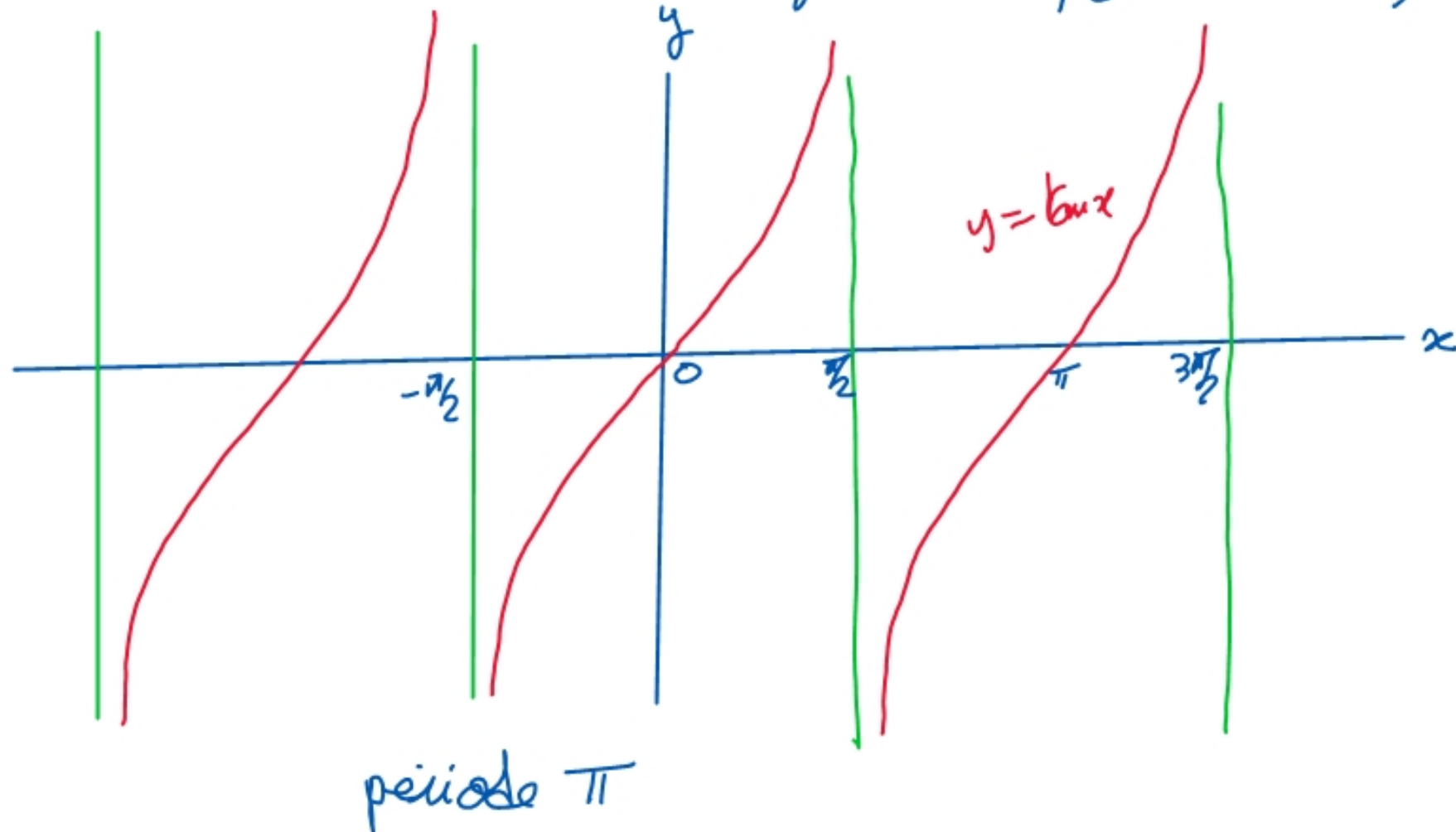
$$e^{i\theta} = \cos \theta + i \sin \theta \quad \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \quad \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

8.4 La fonction tangente

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi \text{ où } k \in \mathbb{Z}$$

$$\text{donc } \mathcal{D}_f = \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi \text{ où } k \in \mathbb{Z} \right\}$$



$$\tan'(x) = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$\left[\tan' x = \left(\frac{\sin x}{\cos x} \right)' = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \right]$$

$$\left(\left(\frac{u}{v} \right)' = \frac{u'v - uv'}{v^2} \right)$$

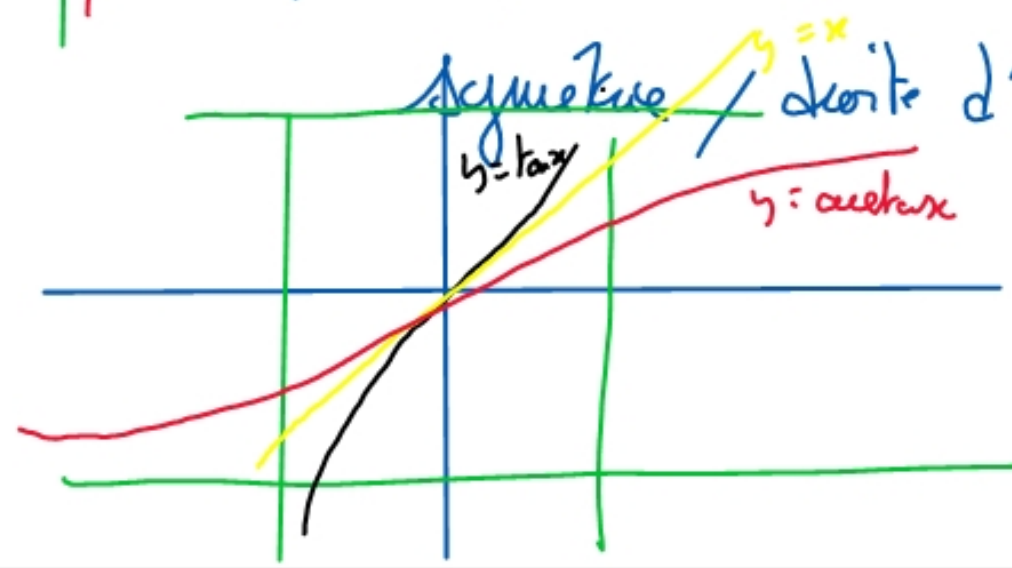
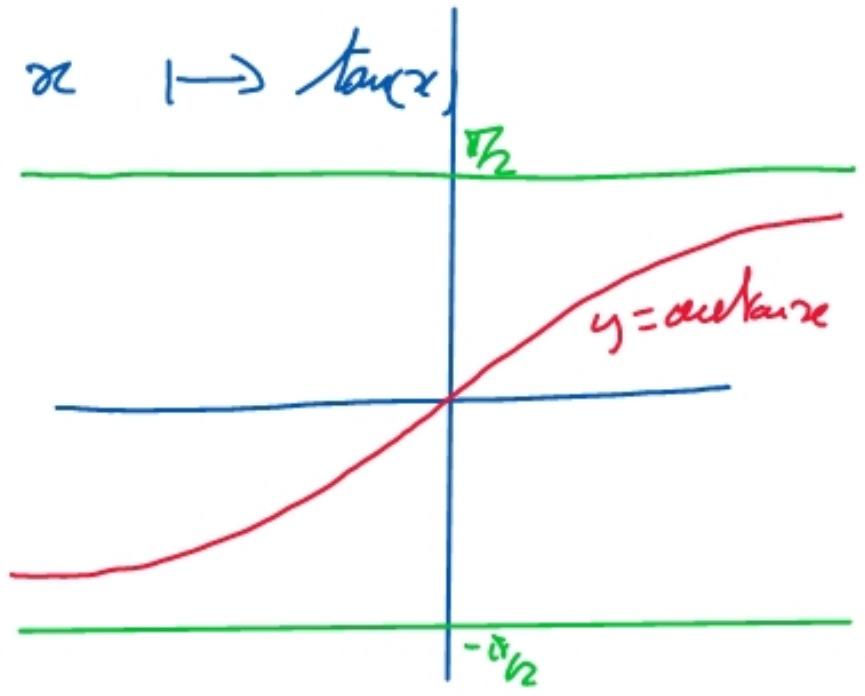
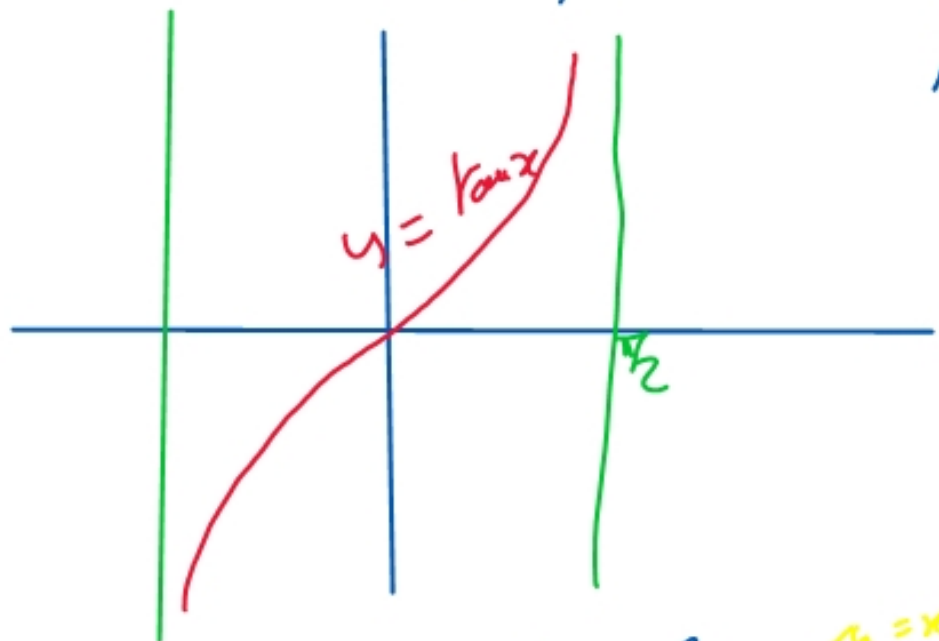
$$= \frac{1}{\cos^2 x}$$

$$= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$= 1 + \tan^2 x \quad \left. \right]$$

La fonction arctan x

c'est la réciproque de \tan | $]-\frac{\pi}{2}; \frac{\pi}{2}[\rightarrow \mathbb{R}$
 $x \mapsto \arctan x$



symétrie / droite d'éq $y = x$

$$\tan(\arctan x) = x, \forall x \in \mathbb{R}$$

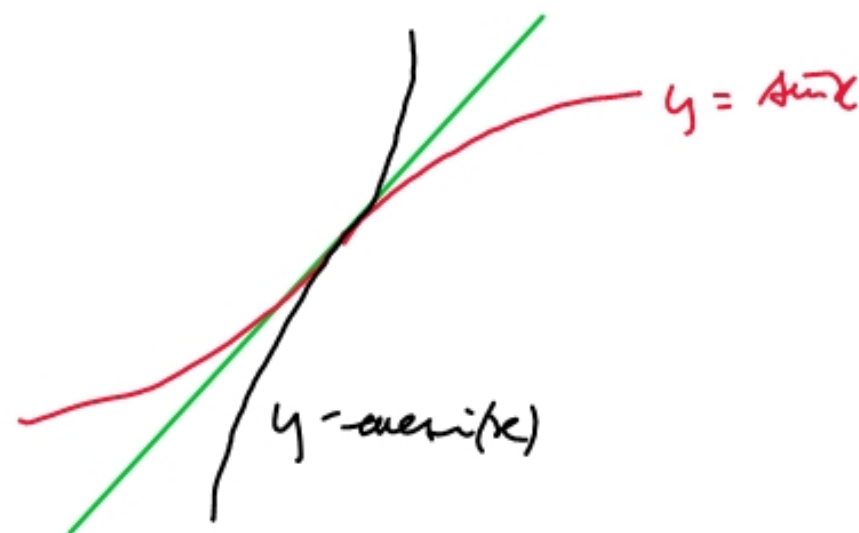
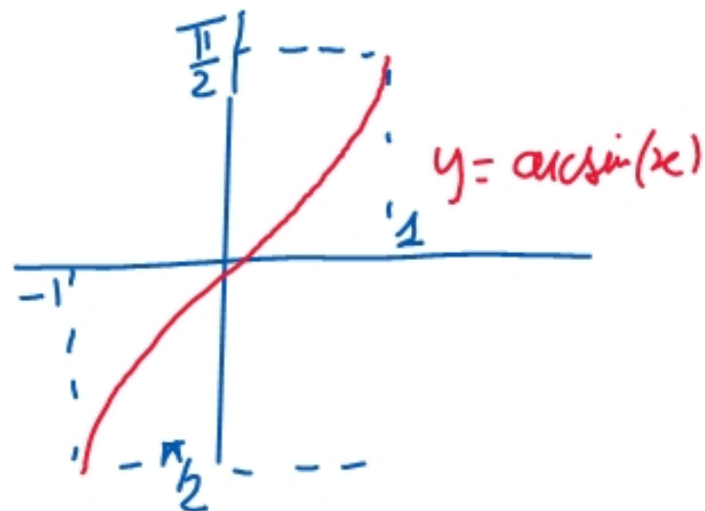
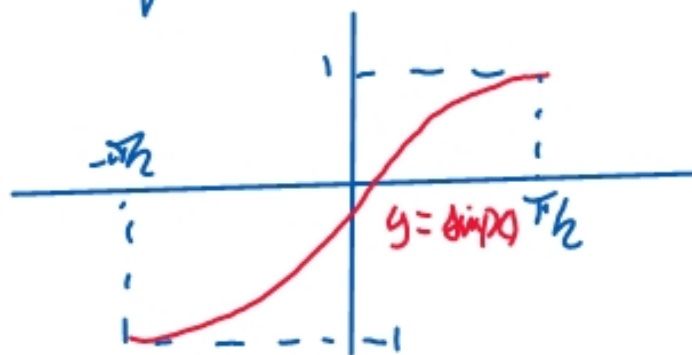
$$\arctan(\tan x) = x, \forall x \in]-\frac{\pi}{2}; \frac{\pi}{2}[$$

$$\arctan' x = \frac{1}{1+x^2}$$

arctan est impaire

$$\arctan(-x) = -\arctan(x)$$

La fonction arcsin(x) C'est la réciproque de \sin / $[-\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow [-1, 1]$
 $x \mapsto \sin(x)$



$$(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$$

6.5 parité, période, ensemble de définition.

def: $f: \mathbb{R} \rightarrow \mathbb{R}$ est paire lorsque $\forall x \in \mathbb{R}, f(-x) = f(x)$

$f: \mathbb{R} \rightarrow \mathbb{R}$ est impaire lorsque $\forall x \in \mathbb{R}, f(-x) = -f(x)$

ex: valeur absolue, \cos, x^2 sont paires

$x^3, \sin(x), \tan(x), \arctan(x)$ sont impaires

ex: parité de $f(x) = \cos(x^3)$?

$$f(-x) = \cos((-x)^3) = \cos(-x^3) = \cos(x^3) = f(x)$$

donc f est paire

ex 2: $f(x) = \frac{x(e^x+1)}{e^x-1}$ parité?

$$f(-x) = \frac{-x(e^{-x}+1)}{e^{-x}-1} \begin{matrix} \times e^x \\ \times e^x \end{matrix} = \frac{-x(1+e^x)}{1-e^x} = \frac{x(1+e^x)}{-(1-e^x)}$$

$$= \frac{x(1+e^x)}{e^x-1} = f(x) \quad \cdot \quad f \text{ est paire}$$

ex 3: $f(x) = x+37$

$$f(1) = 38$$

$$f(-1) = 36$$

$f(-1) \neq f(1)$ donc f n'est pas paire

$f(-1) \neq -f(1)$ donc f n'est pas impaire

) ni paire
ni impaire

[T] Les fonctions paires sont symétriques / l'axe des y

Les fonctions impaires sont symétriques / \circ

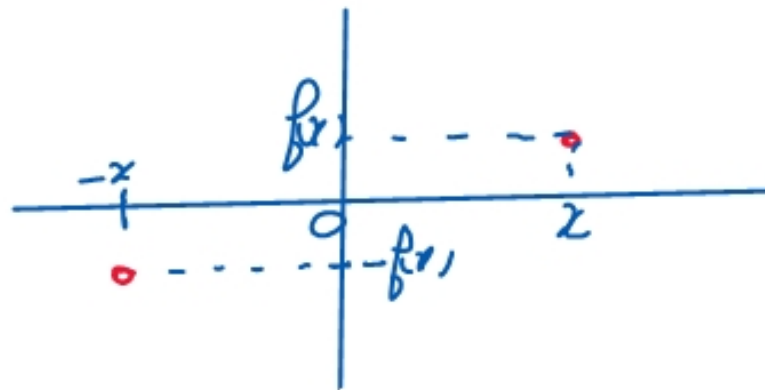
\uparrow
l'origine du repère

fonction
paire



symétrique / oy

fonction
impaire



symétrique / \circ

