

6

recess

2012

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2012 ① $x=y^2$ c'est une parabole

② $\Gamma(t) = (t^2, t)$. On dérive: $\vec{T}(t) = (2t, 1)$

$$\vec{N}(t) = (1, -2t)$$

③ Un vecteur directeur de Γ_A est $\vec{u} = (2a, 1)$, sa pente $\frac{1}{2a}$

$$\Gamma_A: y = \frac{1}{2a}x + \beta$$

Γ_A passe par $A(a^2, a)$

$$a = \frac{1}{2a}a^2 + \beta \Rightarrow \beta = a - \frac{a}{2} = \frac{a}{2} \quad y = \frac{1}{2a}x + \frac{a}{2}$$

$$\Gamma_A: 2ay - x = a^2$$

$$y=0 \Rightarrow x = -a^2 \quad A' = (-a^2, 0)$$

$$\Gamma_B: 2by - x = b^2$$

$$y=0 \Rightarrow x = -b^2 \quad B' = (-b^2, 0)$$

$$\textcircled{4} \quad I = \frac{A+B}{2} = \left(\frac{a^2+b^2}{2}, \frac{a+b}{2} \right)$$

$$\textcircled{5} \quad \begin{cases} 2ay - x = a^2 \\ 2by - x = b^2 \end{cases}$$

$$L_1 - L_2: (2a - 2b)y = a^2 - b^2 \rightarrow y = \frac{a^2 - b^2}{2(a-b)} = \frac{(a-b)(a+b)}{2(a-b)}$$

$$bL_1 - aL_2: (-b + a)x = ba^2 - ab^2 \Rightarrow x = \frac{ba^2 - ab^2}{a-b} = \frac{ab(a-b)}{a-b}$$

$$x = ab$$

$$\textcircled{6} \quad y_I = y_M$$

$$\textcircled{7} \quad y_c = \frac{a+b}{2} \Rightarrow x_c = \left(\frac{a+b}{2}\right)^2 \quad (\text{Rappel: } M(t) = (t^2, t))$$

$$\textcircled{8} \quad \vec{T}\left(\frac{a+b}{2}\right) = \left(2 \times \frac{a+b}{2}; 1\right) = (a+b; 1) \quad (\text{Rappel } \vec{T}(t) = (2t, 1))$$

pente $\frac{1}{a+b}$

$$\vec{AB} = (b^2 - a^2, b - a) \quad \text{pente} = \frac{b-a}{b^2 - a^2} = \frac{\cancel{b-a}}{(\cancel{b-a})(b+a)} = \frac{1}{b+a}$$

\hat{m} pente, deux droites //.

2014 $\textcircled{1}$ $\text{Mat}_{\mathcal{B}}(\mathcal{B}) = \begin{pmatrix} \text{Coord de } f(e_1) & f(e_2) & f(e_3) & f(e_4) \\ \text{dans } \mathcal{B} & \text{dans } \mathcal{B} & \text{dans } \mathcal{B} & \text{dans } \mathcal{B} \end{pmatrix}$

$$f(e_1) = f\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = e_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$f(e_2) = f\left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$f(e_3) = f\left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$f(e_4) = f\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Mat}_{\mathcal{B}}(\mathcal{B}) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{vmatrix} -x & 0 & 0 & 1 \\ 0 & -x & 1 & 0 \\ 0 & 1 & -x & 0 \\ 1 & 0 & 0 & -x \end{vmatrix} \stackrel{\text{dét}}{=} -x \begin{vmatrix} -x & 1 & 0 \\ 1 & -x & 0 \\ 0 & 0 & -x \end{vmatrix} - 1 \begin{vmatrix} 0 & 0 & 1 \\ -x & 1 & 0 \\ 1 & -x & 0 \end{vmatrix}$$

$$= -x(-x) \begin{vmatrix} -x & 1 \\ 1 & -x \end{vmatrix} - 1 \begin{vmatrix} -x & 1 \\ 1 & -x \end{vmatrix}$$

$$= x^2(x^2-1) - (x^2-1) = (x^2-1)(x^2-1) = \underline{(x-1)^2(x+1)^2}$$

$$= x^4 - 2x^2 + 1$$

2 valeurs propres $\lambda = -1$ (de multiplicité 2) et $\mu = 1$ (pareil)

$$\textcircled{5} E_1 = \left\{ \vec{u} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \mid f(\vec{u}) = 1\vec{u} \right\}$$

$$f(\vec{u}) = A \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} d \\ c \\ b \\ a \end{pmatrix} = 1 \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \Rightarrow \begin{cases} a=d \\ b=c \end{cases} \quad \begin{array}{l} 4 \text{ inconnues et 2 eq} \\ \text{donc dimension 2} \end{array}$$

$$\vec{u} = (1, 0, 0, 1) \quad \text{et} \quad \vec{v} = (0, 1, 1, 0) \quad E_1 = \text{Vect}(\vec{u}, \vec{v})$$

$$E_{-1} = \text{Vect} \left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right)$$

$\textcircled{6}$ Yes

car $\textcircled{5}$ le poly caract est scindé et
la dimension des espaces propres est égale à la multiplicité des vpc

⑦ P est la mat de passage de B à $B' = (\vec{u}, \vec{v}, \vec{w}, \vec{z})$

$$P = \begin{pmatrix} \text{coord de} & & & \\ \vec{u} & \vec{v} & \vec{w} & \vec{z} \\ \mathcal{B} & \mathcal{B} & \mathcal{B} & \mathcal{B} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

et $D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ $D = P^{-1}AP$ est la forme de diag de bloc.

Partie B ①: $\text{Mat}_{\mathcal{B}}(g) = \begin{pmatrix} \text{coord de} & & & \\ g(e_1) & g(e_2) & g(e_3) & g(e_4) \\ \text{sur } \mathcal{B} & \text{sur } \mathcal{B} & \text{sur } \mathcal{B} & \text{sur } \mathcal{B} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

\uparrow $g(e_1) = e_4$ \uparrow $g(e_3) = \vec{0}$

② $\begin{vmatrix} -x & 0 & 0 & 0 \\ 0 & -x & 1 & 0 \\ 0 & 1 & -x & 0 \\ 1 & 0 & 0 & -x \end{vmatrix} \stackrel{\text{dapt}}{=} L_1 \quad -x \begin{vmatrix} -x & 1 & 0 \\ 1 & -x & 0 \\ 0 & 0 & -x \end{vmatrix} \stackrel{\text{dapt}}{=} L_3$

$(-x)(-x) | -x \ 1 | = x^2(x+1)(x-1)$

③ 3 valeurs propres, $\alpha = -1$, $\beta = 1$
 mult 2 mult 1 mult 1

④ $E_0 = \left\{ \vec{u} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \mid g(\vec{u}) = \vec{c} \right\} = \text{Ker}(g)$

On trouve $E_0 = \text{Vect}(e_4)$ $N_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$$\textcircled{5} \quad E_1 = \text{Vect} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \quad \text{et} \quad \textcircled{6} \quad E_1 = \text{Vect} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

\uparrow u_1 \uparrow v_1

$\textcircled{7}$ pas diagonalisable car $\dim(E_0) \neq \text{mult}(0)$

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② $M(t) = (t^2, t)$. On dérive: $\vec{T}(t) = (2t, 1)$

$$\vec{N}(t) = (1, -2t)$$

③ Un vecteur directeur de T_A est $\vec{u} = (2a, 1)$, de pente $\frac{1}{2a}$

$$T_A: y = \frac{1}{2a}x + \beta$$

$$T_A \text{ passe par } A(a^2, a) \Rightarrow a = \frac{1}{2a}a^2 + \beta \Rightarrow \beta = a - \frac{a}{2} = \frac{a}{2} \Rightarrow y = \frac{1}{2a}x + \frac{a}{2}$$

$$T_A: 2ay - x = a^2$$

$$y=0 \Rightarrow x = -a^2 \quad A' = (-a^2, 0)$$

$$T_B: 2by - x = b^2$$

$$y=0 \Rightarrow x = -b^2 \quad B' = (-b^2, 0)$$

④ $I = \frac{A+B}{2} = \left(\frac{a^2+b^2}{2}, \frac{a+b}{2} \right)$

⑤ $\begin{cases} 2ay - x = a^2 \\ 2by - x = b^2 \end{cases}$

$$l_1 - l_2: (2a - 2b)y = a^2 - b^2 \Rightarrow y = \frac{a^2 - b^2}{2(a-b)} = \frac{(a-b)(a+b)}{2(a-b)} = \frac{a+b}{2}$$
$$bl_1 - al_2: (-b+a)x = ba^2 - ab^2 \Rightarrow x = \frac{ba^2 - ab^2}{a-b} = \frac{ab(a-b)}{a-b}$$

$x = ab$

⑥ $y_I = y_M$

$$\textcircled{7} \quad y_c = \frac{a+b}{2} \Rightarrow x_c = \left(\frac{a+b}{2}\right)^2 \quad (\text{Rappel: } M(H) = (t^2, t))$$

$$\textcircled{8} \quad \vec{T}\left(\frac{a+b}{2}\right) = \left(2 \times \frac{a+b}{2}; 1\right) \\ = (a+b, 1) \quad \text{pente } \frac{1}{a+b}$$

$$\vec{AB} = (b^2 - a^2, b - a) \quad \text{pente} = \frac{b-a}{b^2 - a^2} = \frac{\cancel{b-a}}{(b-a)(b+a)} = \frac{1}{b+a}$$

in pente, donc droites //

2014

$$\textcircled{1} \quad \text{Mat}_{\mathcal{B}}(\mathcal{P}) = \begin{pmatrix} \text{Coord de} \\ f(e_1) \\ \text{dans } \mathcal{B} & f(e_2) \\ \text{dans } \mathcal{B} & f(e_3) \\ \text{dans } \mathcal{B} & f(e_4) \\ \text{dans } \mathcal{B} \end{pmatrix}$$

$$f(e_1) = f\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = e_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$f(e_2) = f\left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$f(e_3) = e_2 \quad \text{et} \quad f(e_4) = e_1$$

$$\text{Mat}_{\mathcal{B}}(\mathcal{P}) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}
 & \left| \begin{array}{cccc|c} -x & 0 & 0 & 1 & \\ 0 & -x & 1 & 0 & \\ 0 & 1 & -x & 0 & \\ 1 & 0 & 0 & -x & \end{array} \right| \stackrel{\text{dévelop}}{=} -x \left| \begin{array}{ccc|c} -x & 1 & 0 & \\ 1 & -x & 0 & \\ 0 & 0 & -x & \end{array} \right| - 1 \left| \begin{array}{ccc|c} 0 & 0 & 1 & \\ -x & 1 & 0 & \\ 1 & -x & 0 & \end{array} \right| \\
 & \qquad \qquad \qquad \text{col } 3 \qquad \qquad \qquad \downarrow \\
 & = -x(-x) \left| \begin{array}{cc|c} -x & 1 & \\ 1 & -x & \end{array} \right| - 1 \left| \begin{array}{cc|c} -x & 1 & \\ 1 & -x & \end{array} \right| \\
 & = x^2(x^2-1) - (x^2-1) = (x^2-1)(x^2-1) = \underline{(x-1)^2(x+1)^2}
 \end{aligned}$$

2 valeurs propres $\lambda = -1$ (de multiplicité 2) et $\mu = 1$ (simple)

④ $E_{-1} = \left\{ \vec{u} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \mid f(\vec{u}) = -1\vec{u} \right\}$

$$f(\vec{u}) = A \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} d \\ c \\ b \\ a \end{pmatrix} = -1 \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \Leftrightarrow \begin{cases} a = -d \\ b = -c \end{cases} \quad \begin{array}{l} 4 \text{ inconnues et 2 eq} \\ \text{donc dimension 2} \end{array}$$

$$\vec{u} = (1, 0, 0, -1) \quad \text{et} \quad \vec{v} = (0, 1, -1, 0) \quad E_{-1} = \text{Vect}(\vec{u}, \vec{v})$$

ou trouve

⑤ $E_{-1} = \text{Vect} \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right)$
 \vec{w} \vec{x}

⑥ Yes Car ① le poly caract est scindé et
 ② la dim des espaces propres est égale à la multiplicité des valeurs propres caract.

⑦ L est la mat de passage de B à $B' = (\vec{u}, \vec{v}, \vec{w}, \vec{z})$

$$L = \begin{pmatrix} \text{Coord de} & & & \\ \vec{u} & \vec{u} & \vec{v} & \vec{w} \\ \text{dans} & B & B & B \\ B & & & \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{et } D = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$D = L^{-1}AL$ est la formule de changement de base

Exercice B ①: $\text{Mat}_B(A) =$