

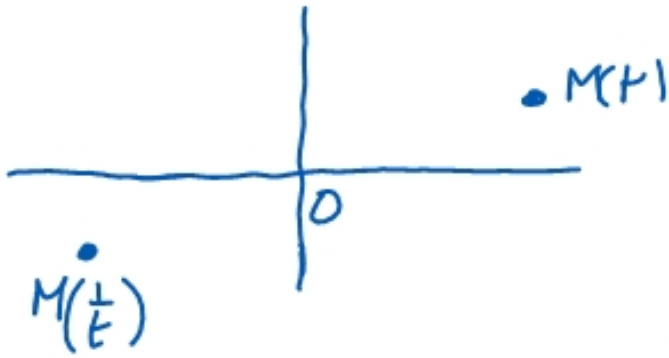
curve

$$\begin{cases} x(t) = \frac{t^2 - 1}{t^2 + 1} \\ y(t) = \ln(t) \end{cases}$$

$$D_f = \mathbb{R}^{+*}$$

$$x\left(\frac{1}{t}\right) = \frac{\frac{1}{t^2} - 1}{\frac{1}{t^2} + 1} \stackrel{\times t^2}{\times t^2} = \frac{1 - t^2}{1 + t^2} = -x(t)$$

$$y\left(\frac{1}{t}\right) = \ln\left(\frac{1}{t}\right) = -\ln(t) = -y(t)$$



Sym/O.

Etude sur]0, 1] + Sym/O

qd t varie de 0 à 1, alors $\frac{1}{t}$ varie de $+\infty$ à 1

on aura tous les pts pour $t \in]0, +\infty[$

$$x'(t) = \frac{2t(t^2 + 1) - 2t(t^2 - 1)}{(t^2 + 1)^2}$$

$$= \frac{4t}{(t^2 + 1)^2} > 0$$

$$y' = \frac{1}{t} > 0$$

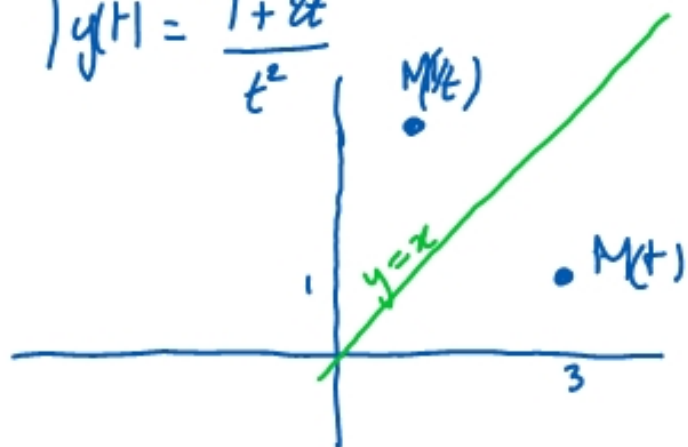
pente en 1: $\frac{y'(1)}{x'(1)} = \frac{1}{\frac{4}{4}} = 1$

t	0	1
x'	+	
x	-1	0
y'	+	
y	$-\infty$	0

asymptote $x=1$ pente 1



$$\begin{cases} x(t) = t^2 + 2t \\ y(t) = \frac{1+2t}{t^2} \end{cases} \quad \mathbb{D} = \mathbb{R}^*$$



$$x\left(\frac{1}{t}\right) = \frac{1}{t^2} + \frac{2}{t} = \frac{1+2t}{t^2} = y(t)$$

$$y\left(\frac{1}{t}\right) = \frac{1 + \frac{2}{t}}{\frac{1}{t^2}} \times \frac{t^2}{t^2} = \frac{t^2 + 2t}{1} = t^2 + 2t = x(t)$$

sim / doite d'eq $y=x$

Estudo em $[-1, 0[\cup]0, 1]$ + sim

si $t \in]0, 1]$, alors $\frac{1}{t} \in [1, +\infty[$

si $t \in [-1, 0[$, alors $\frac{1}{t} \in]-\infty, -1]$

$$x'(t) = 2t + 2 \quad x'(t) = 0 \Leftrightarrow t = -1$$

$$y'(t) = \frac{2t^2 - 2t(1+2t)}{t^4} = \frac{-2t - 2t^2}{t^4} = \frac{-2t(1+t)}{t^4} = \frac{-2(1+t)}{t^3}$$

$$y'(t) = 0 \Leftrightarrow t = -1$$

t	-1	0	1
x'	0	+	+
x	-1	$\rightarrow 0$	$\rightarrow 3$
y'	0	+	-
y	-1	$\rightarrow +\infty$	$\rightarrow 3$

pente 1 I II pente -1
 au $x=0$

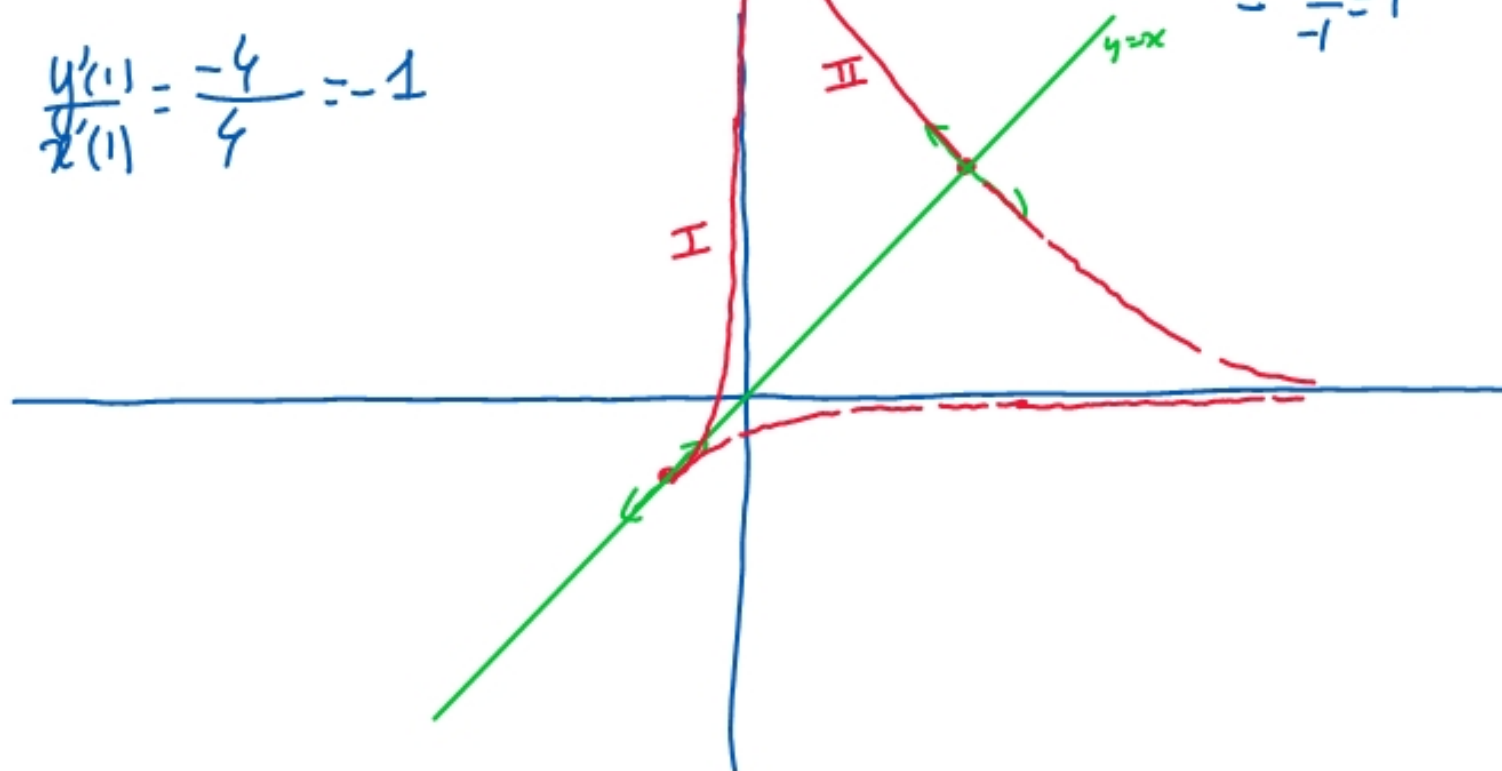
$$y'(0,5) = y'(-\frac{1}{2}) = \frac{-2 \times \frac{1}{2}}{(-\frac{1}{2})^3} > 0$$

$$x'(-\frac{1}{2}) = 2x(-\frac{1}{2}) + 2 = -1 + 2 = 1$$

en $t = -1$: point singulier ($x'(-1) = y'(-1) = 0$)

$$\lim_{t \rightarrow -1} \frac{y'}{x'} = \lim_{t \rightarrow -1} \frac{-2(1+t)}{t^3} = \lim_{t \rightarrow -1} \frac{-2(1+t)}{t^3 \cdot 2(1+t)} = \frac{-1}{-1} = 1$$

en $t = 1$: $\frac{y'(1)}{x'(1)} = \frac{-4}{4} = -1$



2016

$N = (t; -1)$ $A(1, 1)$ $B(-1, -1)$

① éq de la droite (BN) : $\vec{BN} = N - B = (t+1; -t+1)$ pente : $\frac{-t+1}{t+1}$

éq : $y = \frac{-t+1}{t+1}x + b$

Elle passe par B : $-1 = \frac{-t+1}{t+1} \times (-1) + b \Rightarrow b = -1 + \frac{-t+1}{t+1} = \frac{-t-1-t+1}{t+1} = \frac{-2t}{t+1}$

$y = \frac{-t+1}{t+1}x - \frac{2t}{t+1}$

$(t+1)y = (-t+1)x - 2t$

② $\vec{AN} = (t-1; -t-1)$

③ éq de $\Delta \perp \vec{AN}$ passant par A. Un vecteur $\perp \vec{a} (AN)$ est $\vec{u} (t+1, t-1)$

$y = \frac{t-1}{t+1}x + b$ En A : $1 = \frac{t-1}{t+1} + b \Rightarrow b = 1 - \frac{t-1}{t+1} = \frac{t+1-t+1}{t+1} = \frac{2}{t+1}$ $(t+1)y = (t-1)x + 2$

④ $\vec{BN} = (t+1, -t+1)$ Un vect \perp à \vec{BN} est $\vec{u}(t-1, t+1)$

Un vect dit de \perp est $\vec{v}(t+1, t-1)$

$$\vec{u} \cdot \vec{v} = 0 \quad (t-1)(t+1) + (t-1)(t+1) = 0$$
$$\Leftrightarrow 2(t-1)(t+1) = 0 \Leftrightarrow t = \pm 1$$

⑤
$$\begin{cases} (t+1)y + (t-1)x = -2t \\ (t+1)y + (1-t)x = 2 \end{cases}$$
 on cherche x et y . Trop facile!

$$L_1 + L_2 : 2(t+1)y = -2t + 2$$

$$\Rightarrow y = \frac{-t+1}{t+1}$$

$$L_1 - L_2 : 2(t-1)x = -2t - 2$$

$$x = \frac{-t-1}{t-1}$$

$$x = \frac{1+t}{1-t}$$

$$2015 \quad (1) \quad \left\{ \begin{array}{l} x(t) = t \\ y(t) = t^2 \end{array} \right. \text{ ou } \left\{ \begin{array}{l} x(a) = a \\ y(a) = a^2 \end{array} \right. \quad \begin{array}{l} x'(a) = 1 \\ y'(a) = 2a \end{array} \quad \vec{T} = (1, 2a)$$

(2) Une vect \perp à \vec{T} est $(-2a, 1)$ sa pente vaut $-\frac{1}{2a}$

$$y = -\frac{1}{2a}x + b \quad \text{elle passe par } A(a, a^2)$$

$$a^2 = -\frac{1}{2a}a + b \Rightarrow b = a^2 + \frac{1}{2} \quad y = -\frac{1}{2a}x + a^2 + \frac{1}{2}$$

$$2ay = -x + 2a^3 + a$$

$$x + 2ay = 2a^3 + a$$

(3) $B \in \mathbb{D}$: $2ab^2 = -b + 2a^3 + a$ On cherche b

$$2ab^2 + b - 2a^3 - a = 0$$

poly de degré 2 en b .

$$\Delta = 1 + 4 \cdot 2a \cdot (2a^3 + a) = 16a^4 + 8a^2 + 1 = (4a^2 + 1)^2$$

$$b = \frac{-1 \pm (4a^2 + 1)}{4a} = \begin{cases} \frac{4a^2}{4a} = a \\ \frac{-2 - 4a^2}{4a} = -a - \frac{1}{2a} \end{cases}$$