



$$\begin{cases} x(t) = 2 \cos(2t) \\ y(t) = \sin(3t) \end{cases}$$

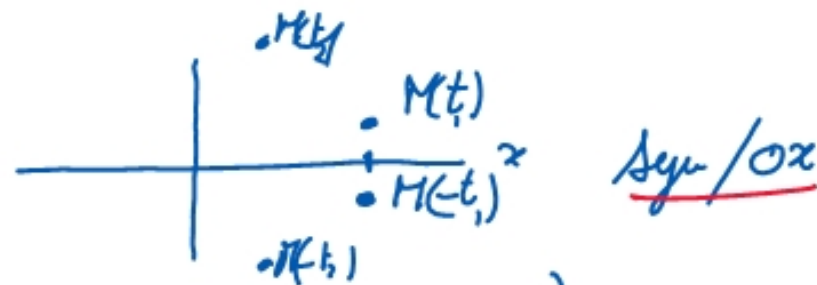
$$x(t+2\pi) = 2 \cos(2(t+2\pi)) = 2 \cos(2t+4\pi) = 2 \cos(2t) = x(t)$$

$$y(t+2\pi) = \sin(3t+6\pi) = \sin(3t) = y(t)$$

donc x et y sont 2π -périodiques

$$x(t) = 2 \cos(2t) = 2 \cos(2t) = x(t)$$

$$y(-t) = \sin(-3t) = -\sin(3t) = -y(t)$$

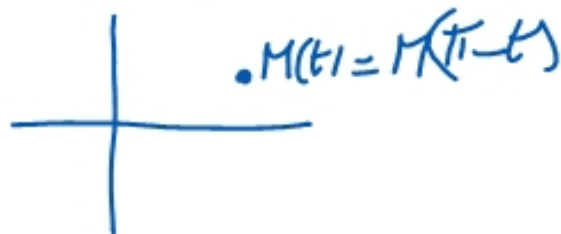


(Si par exemple $M(t) = (3, 1)$ alors $M(-t) = (3, -1)$)

$$x(\pi-t) = 2 \cos(2\pi-2t) = 2 \cos(-2t) = 2 \cos(2t) = x(t)$$

$$y(\pi-t) = \sin(3\pi-3t) = \underbrace{\sin(3\pi)}_0 \cos(3t) - \sin(3t) \underbrace{\cos(3\pi)}_{-1} = \sin(3t) = y(t)$$

$$M(\pi-t) = M(t)$$



Si t varie de 0 à $\frac{\pi}{2}$, alors $\pi - t$ varie de $\pi - 0$ à $\pi - \frac{\pi}{2}$

$M(t)$

$$= M(\pi - t)$$



En dessinant la cbe de 0 à $\frac{\pi}{2}$ on a tous les pts de 0 à π



Si t varie de 0 à π alors $-t$ varie de 0 à $-\pi$

On fait le symétrise / or. On obtient les pts $M(t)$ pour $t \in [\pi, 0]$

On a tous les pts de $-\pi$ à π

$M(t + \pi) = M(t)$, $\forall t$. On a toute la courbe

$$x(t) = 2 \cos(2t)$$

$$x'(t) = -4 \sin(2t)$$

$$x'(t) = 0 \Leftrightarrow \sin(2t) = 0 \Leftrightarrow \underline{2t = k\pi}$$

$$t = \frac{k\pi}{2}$$

$$\sin(x) = 0 \Leftrightarrow x = k\pi$$

$$\cos(x) = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi$$

$$t = 0 \text{ et } \frac{\pi}{2}$$

$$y(t) = \sin(3t)$$

$$y'(t) = 3 \cos(3t)$$

$$y'(t) = 0 \Leftrightarrow \cos(3t) = 0 \Leftrightarrow 3t = \frac{\pi}{2} + k\pi \Leftrightarrow t = \frac{\pi}{6} + \frac{k\pi}{3}$$

t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
x'	0	-	$-2\sqrt{3}$	0
x	2	1	0	-2
y'	3	+	0	-
y	0	1	0	-1

$\text{pointe} = 0,85$
 $\text{pointe} = \frac{3}{0} = \infty \downarrow$

$$k=0: \frac{\pi}{6} \quad k=1: \frac{\pi}{6} + \frac{\pi}{3} = \frac{3\pi}{6} = \frac{\pi}{2}$$

$$\text{en } \frac{\pi}{6} \quad x'(\frac{\pi}{6}) = -4 \sin(\frac{\pi}{3}) = -4 \frac{\sqrt{3}}{2} = -2\sqrt{3}$$

$$y'(\frac{\pi}{6}) = 0$$

$$\text{pointe} = \frac{y'}{x'}(\frac{\pi}{6}) = 0 \Leftrightarrow$$

$$\text{en } 0 \quad x'(0) = 0 \quad y'(0) = 3 \quad \text{pointe} = \frac{3}{0} = \infty \downarrow$$

$$\text{en } \frac{\pi}{2} \quad \text{point singulier car } x'(\frac{\pi}{2}) = y'(\frac{\pi}{2}) = 0$$

en $\frac{0}{0}$.

peut = $\lim_{\frac{\pi}{2}} \frac{y'}{x'} = \lim_{\frac{\pi}{2}} \frac{3 \cos(3t)}{-4 \sin(2t)} = \lim_{\frac{\pi}{2}} \frac{-9 \sin(3t)}{-8 \cos(2t)}$ (Règle de l'Hôpital)

$= \frac{-9 \sin(\frac{3\pi}{2})}{-8 \cos(\pi)} = \frac{9}{8}$

Règle de l'Hôpital :

Si $\begin{cases} f \rightarrow 0 \\ g \rightarrow 0 \end{cases}$ alors $\lim_a \frac{f}{g} = \lim_a \frac{f'}{g'}$

Si $\begin{cases} f \rightarrow \infty \\ g \rightarrow \infty \end{cases}$ alors $\lim_a \frac{f}{g} = \lim_a \frac{f'}{g'}$

chgt $x = t - \frac{\pi}{2}$ $t = x + \frac{\pi}{2}$ $\frac{0}{0}$ $\frac{-}{+}$

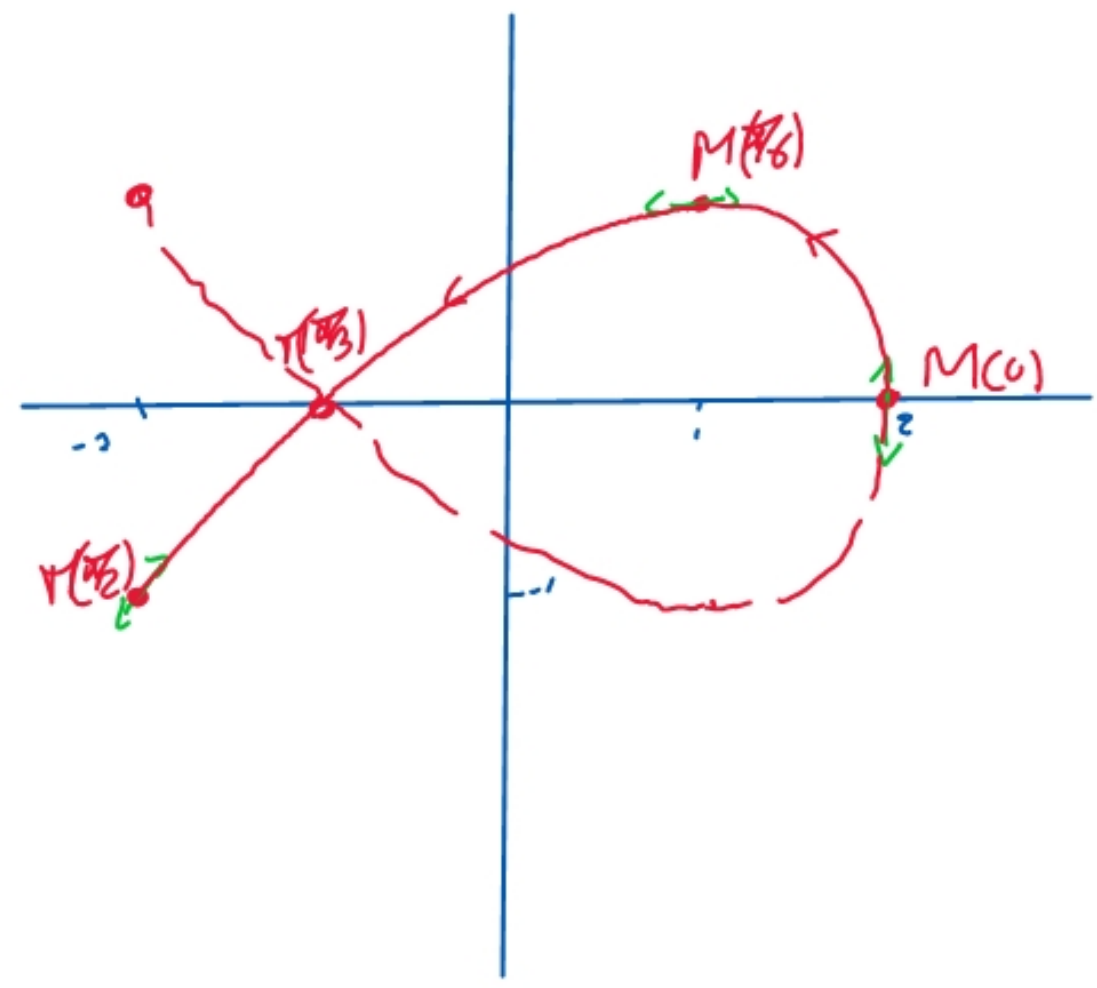
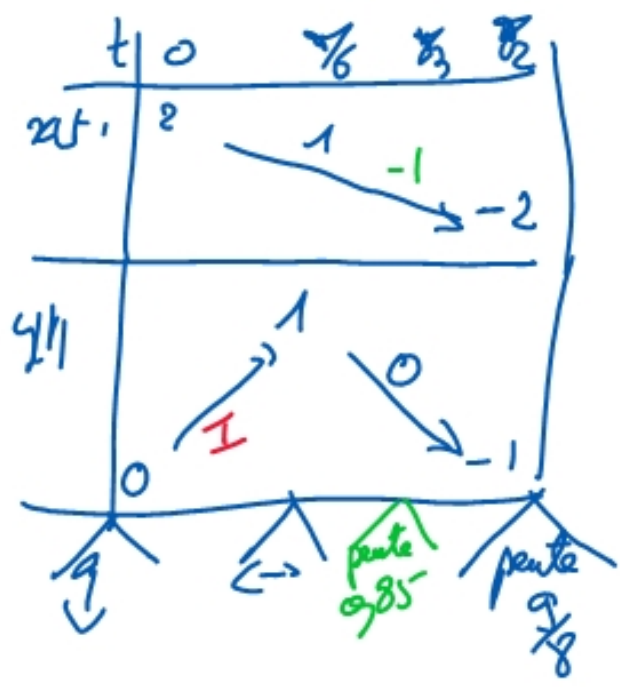
$\frac{3 \cos(3t)}{-4 \sin(2t)} = \frac{3 \cos(3x + \frac{3\pi}{2})}{4 \sin(2x + \pi)} = \frac{3 [\cos(3x) \cos(\frac{3\pi}{2}) - \sin(3x) \sin(\frac{3\pi}{2})]}{-4 \sin(2x)}$

$= \frac{3 \sin(3x)}{4 \sin(2x)} \sim \frac{3 \cdot 3x}{4 \cdot 2x}$

$\cos(x + \pi) = -\cos x$

$\sin(x + \pi) = -\sin x$

$= \frac{9}{8}$



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$$x(t) = \sin^2 t$$

$$y(t) = (1 + \cos t) \sin t$$

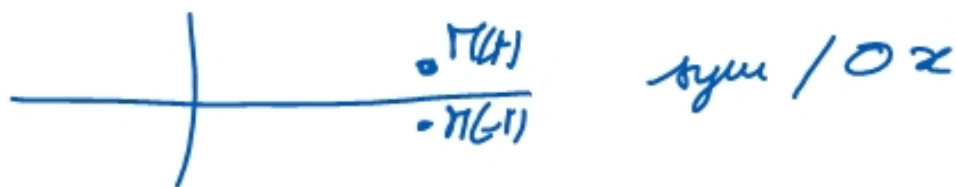
① $x(t+2\pi) = \sin^2(t+2\pi) = \sin^2 t = x(t)$
 $x(t+\pi) = (\sin(t+\pi))^2 = (-\sin t)^2 = \sin^2 t = x(t)$
 $x(t+\frac{\pi}{2}) = \dots \neq x(t)$

↳ la + petite période est π pour $x(t)$

$y(t+2\pi) = \dots = y(t)$
 $y(t+\pi) = (1 - \cos t) \times (-\sin t) \neq y(t)$

↳ la + petite période est 2π pour y .

② $x(-t) = \sin^2(-t) = (-\sin t)^2 = \sin^2 t = x(t)$ } x est pair
 $y(-t) = (1 + \cos(-t)) \sin(-t) = (1 + \cos t) \times (-\sin t) = -y(t)$ } y est impair



③ $x(t) = \sin^2 t$ $x'(t) = 2 \sin t \cos t$ ($u^2' = 2u \cdot u'$)
 x' s'annule \Leftrightarrow $\begin{cases} \sin t = 0 \\ \text{ou} \\ \cos t = 0 \end{cases} \Leftrightarrow \begin{cases} t = k\pi \\ \text{ou} \\ t = \frac{\pi}{2} + k\pi \end{cases}$ entre 0 et 2π : $0, \frac{\pi}{2}, \pi$

$$y'(t) = ((1 + \cos t) \sin t)' = -\sin t \times \sin t + (1 + \cos t) \cos t$$

$$= \underbrace{-\sin^2 t}_{-1} + \cos t + \cos^2 t$$

$$= \cos^2 t - 1 + \cos t + \cos^2 t = \underline{2\cos^2 t + \cos t - 1}$$

$$\cos^2 t + \sin^2 t = 1$$

$$\Rightarrow -\sin^2 t = \cos^2 t - 1$$

On pose $x = \cos t$ $y'(t) = 2x^2 + x - 1$

$$\Delta = 9 \quad x = \frac{-1 \pm 3}{4} = \begin{cases} -1 \\ \frac{1}{2} \end{cases}$$

$$y'(t) = 0 \Leftrightarrow \begin{cases} \cos t = -1 & \text{car} \\ \text{ou} \\ \cos t = \frac{1}{2} \end{cases} \left. \begin{array}{l} t = \pi + 2k\pi \\ l = \pm \frac{\pi}{3} + 2k\pi \end{array} \right) \text{ici en } \frac{\pi}{3} \text{ et } \pi$$

t	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
x'	0	$+\frac{\sqrt{3}}{2}$	0	$-\frac{\sqrt{3}}{2}$	0
x	0	$\frac{3}{4}$	1	$\frac{3}{4}$	0
y'	2	0	-1	-1	0
y	0	$\frac{3\sqrt{3}}{4}$	1	$\frac{\sqrt{3}}{4}$	0

$$x'(\frac{\pi}{3}) = 2 \frac{\sqrt{3}}{2} \frac{1}{2} = \frac{\sqrt{3}}{2} \quad y(\frac{\pi}{3}) = (1 + \frac{1}{2}) \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$$

$$y'(0) = 2 \cdot 1 + 1 - 1 = 2$$

$$x'(\frac{2\pi}{3}) = 2 \frac{\sqrt{3}}{2} (-\frac{1}{2}) = -\frac{\sqrt{3}}{2}$$

$$y'(\frac{\pi}{2}) = 2 \cdot 0 + 0 - 1 = -1$$

$$x(\frac{\pi}{3}) = \frac{3}{4}$$

pende en $\frac{\pi}{3}$: $\frac{0}{\frac{\sqrt{3}}{2}} = 0 \Leftrightarrow y'(\frac{\pi}{3}) = 2 \cdot \frac{1}{4} - \frac{1}{2} - 1 = -1$

en $\frac{\pi}{2}$: $\frac{1}{0} = \infty \uparrow$

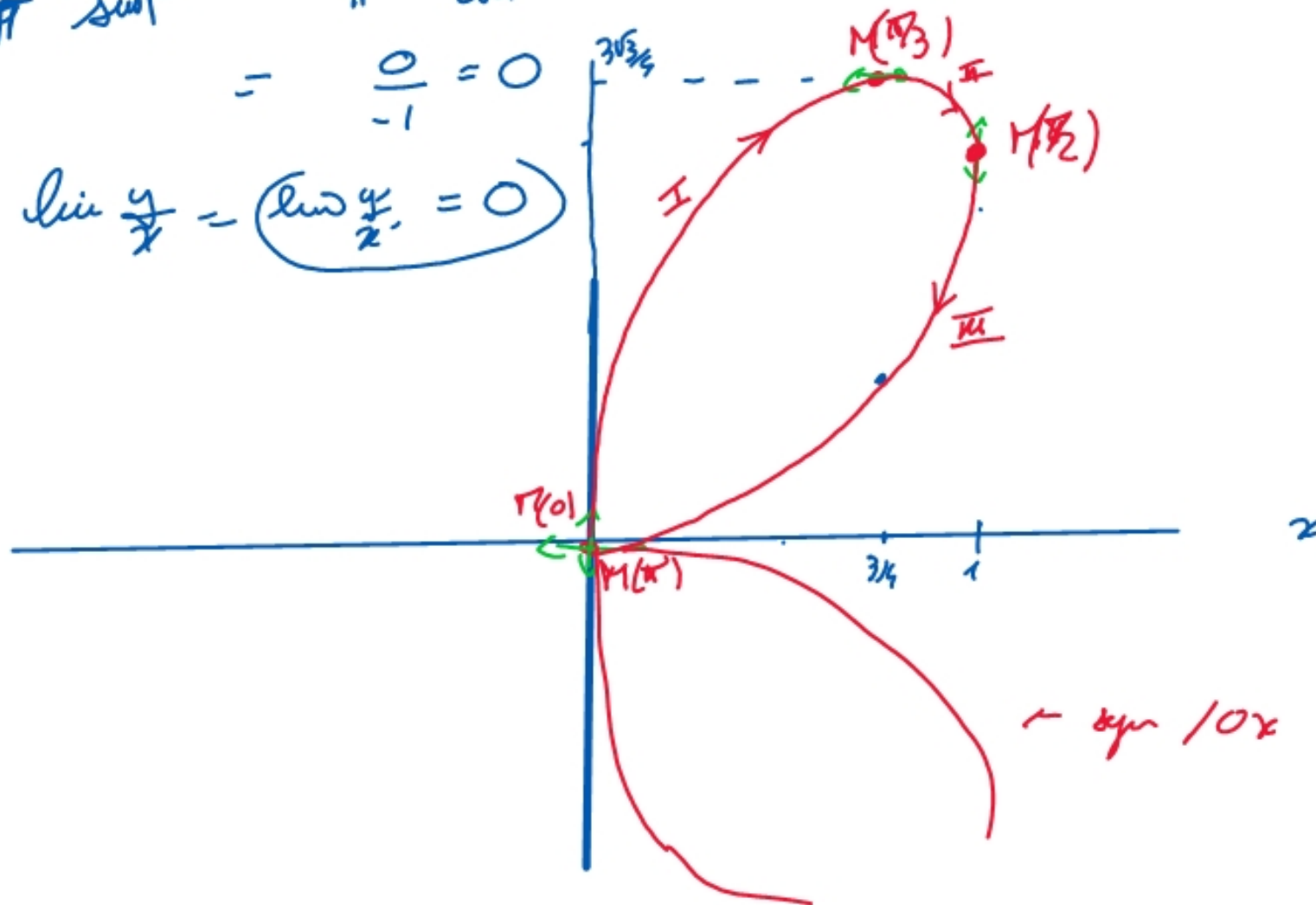
$$y(\frac{\pi}{3}) = (1 - \frac{1}{2}) \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

(5) $\frac{y(t)}{x(t)} = \frac{(1+\cos t) \sin t}{\sin^2 t} = \frac{1+\cos t}{\sin t} \xrightarrow{t \rightarrow 0} \frac{0}{0} = \infty$
 $\xrightarrow{t \rightarrow \pi} \frac{1-1}{0}$ F.I.

$\lim_{t \rightarrow \pi} \frac{1+\cos t}{\sin t} = \lim_{t \rightarrow \pi} \frac{-\sin t}{\cos t}$ (HOSPITAL)

$= \frac{0}{-1} = 0$

$\lim_{t \rightarrow \pi} \frac{y}{x} = \left(\lim_{t \rightarrow \pi} \frac{y}{x} = 0 \right)$



$y'(t)$