

① Somme partielle géométrique

$$\sum_{k=0}^{n} z^k = 1 + z + z^2 + \dots + z^n = \begin{cases} \frac{1-z^{n+1}}{1-z} & \text{qd } z \neq 1 \\ n+1 & \text{qd } z = 1 \end{cases}$$

$$\textcircled{2} \quad 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^{10}} = \frac{1 - \left(\frac{1}{2}\right)^{11}}{1 - \frac{1}{2}} = \frac{1 - \frac{1}{2^{11}}}{\frac{1}{2}} \stackrel{\times 2^{11}}{=} \frac{2^{11} - 1}{2^{10}} = \frac{2047}{1024}$$

③ DL de  $\sin(x)$  à l'ordre  $2n+1$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1})$$

④ Dérivées de  $\tan(x)$   $\tan'(x) = 1 + \tan^2(x) = \frac{1}{\cos^2(x)}$

⑤ Def de la matrice de passage de la base  $B$  à la base  $B'$

$$P_{B \rightarrow B'} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \text{ où } B' = (\vec{u}, \vec{v}) \text{ et } B = (\vec{u}, \vec{v})$$

$$\textcircled{1} f(x) = 3x^n e^{-x^2} - 1$$

$x > 0$

$$f'(x) = 3nx^{n-1}e^{-x^2} + 3x^n e^{-x^2} \times (-2x)$$

$$= 3nx^{n-1}e^{-x^2} - 6x^{n+1}e^{-x^2}$$

$$= 3x^{n-1}e^{-x^2} \left[ n - 2x^2 \right]$$

$\uparrow$        $\uparrow$        $\underbrace{\hspace{2cm}}$   
 $> 0$      $> 0$       du signe de  $n - 2x^2$

$$x(t) = \frac{t^2 + 1}{t}$$

$$x'(t) = \frac{2t \cdot t - (t^2 + 1) \cdot 1}{t^2} = \frac{t^2 - 1}{t^2} = \frac{(t-1)(t+1)}{t^2}$$

$$y(t) = \frac{5t+4}{t(t-1)} = \frac{5t+4}{t^2-t}$$

$$y'(t) = \frac{5(t^2-t) - (5t+4)(2t-1)}{(t^2-t)^2}$$

$$y'(t) = \frac{5t^2 - 5t - (10t^2 - 5t + 8t - 4)}{(t^2-t)^2} = \frac{-5t^2 - 8t + 4}{(t^2-t)^2}$$

$$\Delta = 64 + 80 = 144 = 12^2$$

$$2 \text{ racines : } \frac{8 \pm 12}{-10} = \left\{ \begin{array}{l} -2 \\ \frac{2}{5} \end{array} \right.$$

$$y'(t) = \frac{-5(t+2)(t-\frac{2}{5})}{(t^2-t)^2} = \frac{-5(t+2)(t-\frac{2}{5})}{t^2(t-1)^2}$$

$$t^2-t = t(t-1)$$



$$y(t) = \frac{\sin^3(t)}{\cos(t)}$$

$$(u^3)' = 3u'u^2$$

$$(\sin^3 t)' = 3\cos t \sin^2 t$$

$$y'(t) = \frac{3\cos t \sin^2 t \cdot \cos t - (-\sin t) \sin^3 t}{\cos^2 t} = \frac{3\cos^2 t \sin^2 t + \sin^4 t}{\cos^2 t} > 0$$

$$= \frac{\sin^2 t (3\cos^2 t + \sin^2 t)}{\cos^2 t}$$

$$\sin^2 + \cos^2 = 1 \Rightarrow \sin^2 = 1 - \cos^2$$

$$= \frac{\sin^2 t (2\cos^2 t + 1)}{\cos^2 t}$$

$$f(x) = \frac{\sin x}{2 - \cos x}$$

$$f'(x) = \frac{\cos x (2 - \cos x) - \sin x \sin x}{(2 - \cos x)^2} = \frac{2\cos x - \cos^2 x - \sin^2 x}{(2 - \cos x)^2}$$

$$= \frac{2\cos x - 1}{(2 - \cos x)^2}$$

$$N = \begin{pmatrix} 0 & 0 & q \\ 1 & 0 & p \\ 0 & 1 & 0 \end{pmatrix}$$

$$\textcircled{11} \quad \mathcal{P}_N(x) = \begin{vmatrix} -x & 0 & q \\ 1 & -x & p \\ 0 & 1 & -x \end{vmatrix} \begin{array}{l} \text{d'apt} / \\ \text{Col}_1 \end{array} \quad -x \begin{vmatrix} -x & p \\ 1 & -x \end{vmatrix} - 1 \begin{vmatrix} 0 & q \\ -x & p \end{vmatrix} + 0 \begin{vmatrix} 0 & q \\ 0 & p \end{vmatrix}$$

$$= -x(x^2 - p) - (-q) = \underline{-x^3 + px + q}$$

Il que

si  $\mathcal{P}(x) = (x - x_0)^2 Q(x)$  alors  $x_0$  est une racine de  $\mathcal{P}'(x)$

$$\mathcal{P}'(x) = 2(x - x_0)Q(x) + (x - x_0)^2 Q'(x) = (x - x_0) [2Q + (x - x_0)Q']$$

$$\mathcal{P}'(x_0) = 0 + 0 \quad \text{donc } x_0 \text{ est racine de } \mathcal{P}'$$

$\textcircled{12}$   $\mathcal{P}_N(x) = -x^3 + px + q$  a une racine double  $a$

$$\mathcal{P}_N(a) = 0 \quad \left. \begin{array}{l} -a^3 + pa + q = 0 \\ -3a^2 + p = 0 \end{array} \right\}$$

$$\mathcal{P}'_N(a) = 0 \quad \left. \begin{array}{l} -3a^2 + p = 0 \end{array} \right\}$$

$$\Rightarrow p = 3a^2 \quad \begin{array}{l} \nearrow \\ \nearrow \end{array}$$

$$4p^3 = 4 \cdot 27a^6$$

$$-a^3 + 3a^2 \cdot a + q = 0 \quad 2a^3 + q = 0$$

$$q = -2a^3$$

$$27q^2 = 27 \cdot 4a^6$$

$$\boxed{4p^3 = 27q^2}$$

$\underbrace{4p^3 = 27q}_{\text{pair}} \quad \underbrace{27q}_{\text{imp}}$

$p=1 \quad q=1 \quad p=2 \quad q=1 \quad p=3$

$q$  est pair  $q=2x?$  ou essai 2

$4p^3 = 27 \cdot 4 \Rightarrow p=3$

$N = \begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix}$

$P_N(x) = -x^3 + 3x + 2$  (21 Rubik)

$P_N(-1) = 1 - 3 + 2 = 0$  -1 est racine de  $P_N$ .

$$\begin{array}{r} -x^3 + 3x + 2 \\ \underline{-(-x^3 - x^2)} \\ x^2 + 3x + 2 \\ \underline{-(x^2 + x)} \\ 2x + 2 \\ \underline{2x + 2} \\ 0 \end{array}$$

$\frac{x+1}{-x^2 + x + 2} \quad \Delta = 9 \quad 2 \text{ rac. } -1 \text{ et } 2$

-1 est racine de multiplicité 2  
 2 ——— simple

J'appelle  $B = (v_1, v_2)$  la base canonique de  $\mathbb{R}^2$ . J'appelle  $f \dots$

Calcul de  $E_{-1} = \left\{ \vec{u} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 / f(\vec{u}) = -\vec{u} \right\}$ .

$$f(\vec{u}) = -\vec{u} \Leftrightarrow \begin{cases} 2z = -x \\ x + 2z = -y \\ y = -z \end{cases} \Leftrightarrow \begin{cases} x + 2z = 0 \\ x + y + 2z = 0 \\ y + z = 0 \end{cases} \Leftrightarrow \begin{cases} x + 2z = 0 \\ y + z = 0 \\ 0 = 0 \end{cases}$$

donc  $E_{-1} = \text{Vect} \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$

Calcul de  $E_2 = \text{Vect} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

Q25:  $N$  n'est pas diagonalisable car la dimension de  $E_{-1}$  ( $=1$ ) n'est pas égale à la multiplicité de la valeur propre  $-1$

Q26:  $M = N + I = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} = M = M_{\mathcal{B}}(g)$

$\vec{u}$

$$\vec{u} = (-2, -1, 1) \quad \vec{v} = (1, 2, 1) \quad \vec{w} = (3, -3, 0)$$

$$M = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} = \text{Mat}_{\mathcal{B}}(g)$$

$$g(\vec{u}) = M \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{\mathcal{B}} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{\mathcal{B}'}$$

$$g(\vec{v}) = M \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}_{\mathcal{B}} = 3 \times \vec{v} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}_{\mathcal{B}'}$$

$$g(\vec{w}) = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}_{\mathcal{B}}$$

$$\boxed{-2\vec{u} - \vec{v}} = -2 \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} = g(\vec{w}) = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}_{\mathcal{B}'}$$

$$\text{Mat}_{\mathcal{B}}(g) = \begin{pmatrix} 0 & 0 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} \textcircled{1} \dot{x}'(t) = -2z'(t) \\ \textcircled{2} \dot{y}'(t) = 3y'(t) - z'(t) \end{cases}$$

$$y' = 0 \Rightarrow y = \text{cte}$$

$$\textcircled{3} \dot{z}'(t) = 0 \quad \dot{z}'(t) = A$$

$$\textcircled{1} \dot{x}' = -2A \Rightarrow \dot{x}'(t) = -2At + B$$

$$\textcircled{2} \dot{y}'(t) = 3y' - A \quad \dot{y}' - 3y' = -A$$

Hom:  $y' = C e^{3t}$

Sol part:  $y_p = a \quad y'_p = 0 \Rightarrow y' - 3y = -3a = -A$   
 $a = \frac{A}{3}$

Sol to the complete:  $y' = C e^{3t} + \frac{A}{3}$

$$\textcircled{20} X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, X' = \begin{pmatrix} -2 & 1 & 3 \\ -1 & 2 & -3 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$





$$a_{n+1} = \frac{-a_n}{(n+1)(2n+1)} \quad \text{pour } n=0: \quad a_1 = \frac{-a_0}{1 \cdot 1} = -a_0$$

$$\text{pour } n=1: \quad a_2 = \frac{-a_1}{2 \cdot 3} = \frac{a_0}{2 \cdot 3}$$

$$\text{pour } n=2: \quad a_3 = \frac{-a_2}{3 \cdot 5} = \frac{-a_0}{2 \cdot 3 \cdot 3 \cdot 5} = \frac{-a_0 \cdot 2^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = \frac{-a_0 2^3}{6!}$$

$$\text{pour } n=3: \quad a_4 = \frac{-a_3}{4 \cdot 7} = \frac{a_0}{2 \cdot 3 \cdot 3 \cdot 4 \cdot 5 \cdot 7} = \frac{a_0 2^4}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}$$

$$a_5 = \frac{-a_4}{5 \cdot 9} = \frac{-a_0 2^5}{2 \cdot 3 \cdot 3 \cdot 4 \cdot 5 \cdot 7 \cdot 5 \cdot 9} = \frac{-a_0 2^5}{10!}$$

$\times 2$ 
 $\times 2$ 
 $\times 8$

Il semble que  $a_n = \frac{(-1)^n a_0 2^n}{(2n)!}$

$$y(x) = a_0 \sum_{n=0}^{+\infty} \frac{(-1)^n 2^n}{(2n)!} x^n = a_0 \sum_{n=0}^{+\infty} \frac{(-1)^n (2x)^n}{(2n)!} = a_0 \sum_{n=0}^{+\infty} \frac{(-2x)^n}{(2n)!}$$

on le vérifie par récurrence

$$f(x) = \sum_0^{+\infty} \frac{(-1)^n (2x)^n}{(2n)!} = \sum_0^{+\infty} \frac{(-2x)^n}{(2n)!}$$

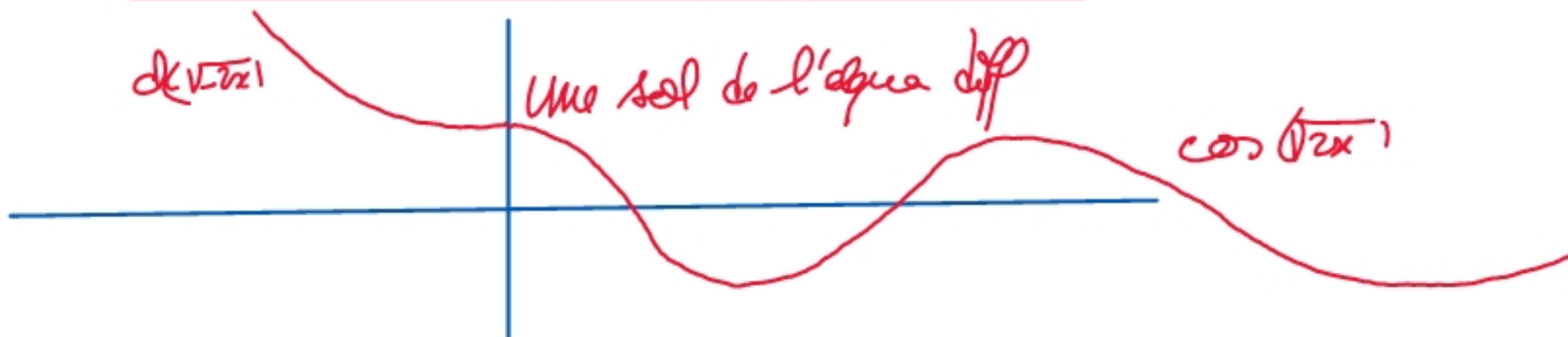
$$\cos(x) = \sum_0^{+\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$f(x) = \sum_0^{+\infty} \frac{(-1)^n \sqrt{2x}^{2n}}{(2n)!} \quad \text{qd } x \geq 0$$

$$\operatorname{ch}(x) = \sum_0^{+\infty} \frac{x^{2n}}{(2n)!}$$

$$f(x) = \sum_0^{+\infty} \frac{(\sqrt{-2x})^{2n}}{(2n)!} \quad \text{qd } x \leq 0$$

$$f(x) = \begin{cases} \cos(\sqrt{2x}) & \text{qd } x > 0 \\ \operatorname{ch}(\sqrt{-2x}) & \text{qd } x < 0 \end{cases}$$



Montrons par récurrence que  $a_n = \frac{(-1)^n 2^n}{(2n)!} a_0$

(sachant que  $a_{n+1} = \frac{-1}{(n+1)(2n+1)} a_n$ )

Pour  $n=0$ .  $a_1 = -\frac{1}{1 \cdot 1} a_0 = \frac{(-1)^1 2^1}{2!} a_0$  OK

Hérédité: Je prends un  $n \geq 0$ . Je suppose que  $a_n = \frac{(-1)^n 2^n}{(2n)!} a_0$

$$a_{n+1} = -\frac{1}{(n+1)(2n+1)} \cdot \frac{(-1)^n 2^n}{(2n)!} a_0 \quad \begin{matrix} \times 2 \\ \times 2 \end{matrix}$$

$$a_{n+1} = \frac{(-1)^{n+1} 2^{n+1}}{(n+1)(2n+1)(2n)! \cdot 2} a_0 = \frac{(-1)^{n+1} 2^{n+1}}{(2n+1)!(2n+2)} a_0 = \frac{(-1)^{n+1} 2^{n+1}}{(2n+2)!} a_0$$

OK!

Conclusion: D'après le principe de récurrence:  $\forall n \in \mathbb{N} \quad a_n = \frac{(-1)^n 2^n}{(2n)!} a_0$