

els

$$x(t) = \frac{t}{t-2}$$

$$D_f = \mathbb{R} - \{-2; 2\}$$

$$x(-t) = \frac{-t}{-t-2} = \frac{t}{t+2} \neq \frac{x(t)}{-x(t)} \text{ ni paire ni impaire}$$

$$y(t) = \frac{8}{t^2-4} = \frac{8}{(t-2)(t+2)}$$

$$y(-t) = y(t) \quad y \text{ est paire}$$

$$x'(t) = \frac{1(t-2) - t \cdot 1}{(t-2)^2} = \frac{-2}{(t-2)^2} < 0$$

$$y'(t) = (8(t^2-4)^{-1})' = -8(t^2-4)^{-2} \cdot 2t = \frac{-16t}{(t^2-4)^2}$$

$$y' = 0 \Leftrightarrow t = 0$$

$$y'(t) = \left(\frac{8}{t^2-4}\right)' = \frac{0(t^2-4) - 8 \cdot 2t}{(t^2-4)^2} = \frac{-16t}{(t^2-4)^2}$$

$$x' = \frac{-2}{(t-2)^2}$$

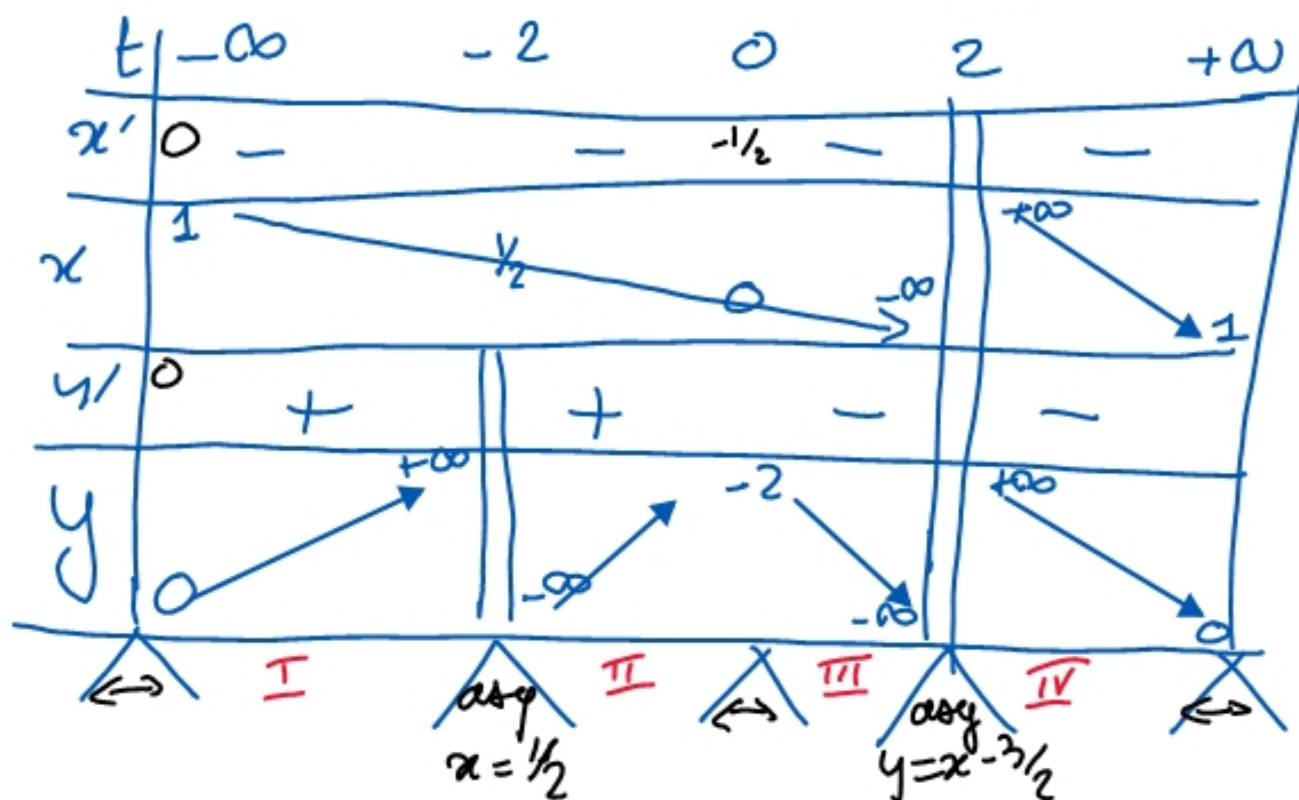
$$y' = \frac{-16t}{(t^2-4)^2}$$

$$x(t) = \frac{t}{t-2}$$

$$x(-2) = \frac{-2}{-4} = \frac{1}{2} \quad x(0) = 0$$

$$y(t) = \frac{8}{t^2-4}$$

$$\lim_{t \rightarrow \infty} x(t) = 1$$



$$\lim_{\infty} y = 0$$

$$y(0) = \frac{8}{-4} = -2$$

EM-? : $x \rightarrow \frac{1}{2}$ $y \rightarrow \infty$
asymptote d'eq $x = \frac{1}{2}$

$$EM0 : x'(0) = \frac{-2}{4} = -\frac{1}{2}$$

$$y'(0) = 0$$

$$\vec{T}(0) = (-\frac{1}{2}; 0) \text{ tpe horiz}$$

qd $t \rightarrow \infty$ A(1,0). pente?

$$\frac{y'}{x'} = \frac{-16t}{(t^2-4)^2} \cdot \frac{(t-2)^2}{-2} \underset{t \rightarrow \infty}{\sim} \frac{-16t^3}{-2t^4} = \frac{8}{t} \rightarrow 0 \text{ tpe horizontale}$$

qd $t \rightarrow 2$: $x \rightarrow \infty$ et $y \rightarrow \infty$ asymptote ?

$$\frac{y}{x} = \frac{8}{t^2-4} \cdot \frac{t-2}{t} = \frac{8 \cancel{(t-2)}}{\cancel{(t-2)}(t+2)t} \xrightarrow{t \rightarrow 2} \frac{8}{4 \cdot 2} = 1 = a$$

$$y - ax = \frac{8}{t^2-4} - \frac{t}{t-2} = \frac{8}{(t-2)(t+2)} - \frac{t}{t-2} = \frac{8 - t(t+2)}{(t-2)(t+2)}$$

$$= \frac{-t^2 - 2t + 8}{(t-2)(t+2)} \xrightarrow{t \rightarrow 2} \frac{0}{0} \quad -t^2 - 2t + 8 = (t-2)(-t-4)$$

$$= \frac{\cancel{(t-2)}(-t-4)}{\cancel{(t-2)}(t+2)} \xrightarrow{t \rightarrow 2} \frac{-6}{4} = -\frac{3}{2}$$

any : $y = x - \frac{3}{2}$

(2) coord de $M\left(\frac{4}{t}\right)$ $x(t) = \frac{t}{t-2}$ $y(t) = \frac{8}{t^2-4}$

$$x\left(\frac{4}{t}\right) = \frac{\frac{4}{t}}{\frac{4}{t}-2} \times \frac{t}{t} = \frac{4}{4-2t} = \frac{2}{2-t}$$

$$M\left(\frac{4}{t}\right) = \left(\frac{2}{2-t}, \frac{2t^2}{4-t^2}\right)$$

$$y\left(\frac{4}{t}\right) = \frac{8}{\left(\frac{4}{t}\right)^2-4} \times \frac{t^2}{t^2} = \frac{8t^2}{16-4t^2} = \frac{4 \cdot 2t^2}{4(4-t^2)}$$

Moyenne de $M(t)$ et $M\left(\frac{4}{t}\right)$: $I = \frac{M(t) + M\left(\frac{4}{t}\right)}{2}$

$$\frac{x(t) + x\left(\frac{4}{t}\right)}{2} = \left(\frac{t}{t-2} + \frac{2}{2-t}\right) \frac{1}{2} = \left(\frac{t-2}{t-2}\right) \cdot \frac{1}{2} = \frac{1}{2}$$

$$\frac{y(t) + y\left(\frac{4}{t}\right)}{2} = \frac{1}{2} \left(\frac{8}{t^2-4} + \frac{2t^2}{4-t^2}\right) = \frac{1}{2} \left(\frac{8-2t^2}{t^2-4}\right) = \frac{1}{2} \frac{2(4-t^2)}{t^2-4} = -1$$

Le milieu de $M(t)$ et $M\left(\frac{4}{t}\right)$ est $B\left(\frac{1}{2}; -1\right)$. Il y a une asymptote / B.

$$A = \int_{x=2}^{x=3} y \, dx = \int_{t=4}^{t=3} \frac{8}{t-4} \cdot \left(\frac{-2}{(t-2)^2} dt \right)$$

$$x = \frac{t}{t-2}$$

$$x' = \frac{dx}{dt} = \frac{-2}{(t-2)^2} \Rightarrow dx = \frac{-2}{(t-2)^2} dt$$

$$= \int_3^4 \frac{16}{(t-4)(t-2)^2} dt = \int_3^4 \frac{16}{(t-2)^3(t+2)} dt$$

$$\frac{16}{(t-2)^3(t+2)} = \frac{E(t)}{0} + \left(\frac{a}{t-2} + \frac{b}{(t-2)^2} + \frac{c}{(t-2)^3} + \frac{d}{t+2} \right)$$

$$c = \frac{16}{t+2} \Big|_{t=2} = \frac{16}{4} = 4$$

$$d = \frac{16}{(t-2)^3} \Big|_{t=-2} = \frac{16}{-16 \cdot 4} = -\frac{1}{4}$$

$$\frac{16}{(t-2)^3(t+2)} - \frac{4}{(t-2)^3} = \frac{16 - 4(t+2)}{(t-2)^3(t+2)} = \frac{-4t + 8}{(t-2)^3(t+2)} = \frac{-4(t-2)}{(t-2)^3(t+2)} = \frac{-4}{(t-2)^2(t+2)}$$

$$\frac{-4}{(t-2)^2(t+2)} = \frac{a}{t-2} + \frac{b}{(t-2)^2} + \frac{-1/4}{t+2}$$

$$b = \frac{-4}{t+2} \Big|_{t=2} = \frac{-4}{4} = -1. \text{ On knowe } \frac{1}{4} \text{ pada}$$

$$\int_3^4 \frac{1}{t-2} dt = \left[\ln |t-2| \right]_3^4 = \ln(2) - \ln(1)$$

$$\int \frac{1}{(t-2)^2} dt = \int (t-2)^{-2} dt$$

$$= \frac{(t-2)^{-1}}{-1} = \frac{-1}{t-2}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$$

$$\int \frac{1}{(t-2)^3} dt = \int (t-2)^{-3} dt = \frac{(t-2)^{-2}}{-2} = -\frac{1}{2(t-2)^2}$$

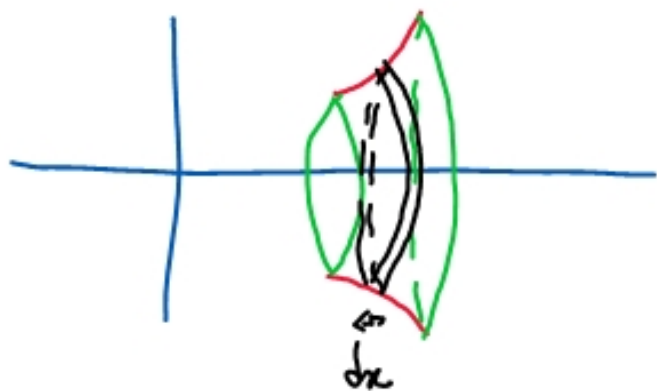
$$\int_3^4 \frac{1}{t+2} dt = \ln |t+2|$$

$$(7) L = \int_{t=3}^{t=4} \sqrt{x^2 + y'^2} dt = \int_3^4 \sqrt{\left(\frac{-2}{(t-2)^2}\right)^2 + \left(\frac{16t}{(t-2)^2(t+2)^2}\right)^2} dt$$

$$(8) \int_{x=\phi}^{x=3} y dx = \int_{t=+\infty}^{t=3} \frac{-16}{(t^2-4)(t-2)^2} dt \quad \text{pb en } +\infty.$$

$$= \int_3^{+\infty} \frac{+16}{\dots}$$

$$\frac{16}{(t^2-4)(t-2)^2} \xrightarrow{+\infty} \frac{16}{t^4} \quad \int \frac{1}{t^4} dt \text{ (Riemann)} \Rightarrow \int y dx \text{ CV (T ds U)}$$



$$dV = \pi y^2 dx$$

$$dV = \pi \frac{64}{(t^2-4)^2} \frac{-2}{(t-2)^2} dt$$

$$V = \int_{t=4}^{t=3} \pi \frac{-128}{(t^2-4)^2(t-2)^2} dt$$

III Développement en Séries Entières des fonctions classiques.

$$\frac{1}{1-x} = \sum_{n=0}^{+\infty} x^n, \quad \forall x \in]-1; 1[$$

Rayon 1

$$\text{ex 1: } \sum_{n=0}^{+\infty} \left(\frac{1}{5}\right)^n = \sum_{n=0}^{+\infty} \left(\frac{1}{5}\right)^n - \left(1 + \frac{1}{5} + \frac{1}{5^2}\right)$$

$$\frac{1}{5^3} + \frac{1}{5^4} + \dots$$

$$= \frac{1}{1 - \frac{1}{5}} - \frac{25 + 5 + 1}{25} = \frac{5}{4} - \frac{31}{25}$$

$$= \frac{5 \cdot 25 - 31 \cdot 4}{100} = \frac{125 - 124}{100} = \frac{1}{100}$$

$$\text{ex 2: } \sum_{n=0}^{+\infty} 5^n = \frac{1}{4-5} - \frac{1}{4} + \infty \quad (\sum 5^n \text{ Div})$$

$$\frac{1}{1+x} = \sum_{n=0}^{+\infty} (-1)^n x^n \quad \forall x \in]-1, 1[$$

on a remplacé x
par $-x$ dans $\frac{1}{1-x}$

on intègre $\frac{1}{1-x} = \sum_{n=0}^{+\infty} x^n$: $-\ln(1-x) = \sum_{n=0}^{+\infty} \frac{x^{n+1}}{n+1} = \sum_{n=1}^{+\infty} \frac{x^n}{n}$
 $x \in]-1, 1[$

$$\ln(1-x) = -\sum_{n=1}^{+\infty} \frac{x^n}{n}, \quad \forall x \in]-1, 1[$$

$$\ln(1-x) = -x - \frac{x^2}{2} \dots - \frac{x^n}{n} - \dots$$

$$\ln(1+x) = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1} x^n}{n}, \quad \forall x \in]-1, 1[$$

on a remplacé x par $-x$
et : $-(-1)^n = (-1)^{n+1}$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n+1} \frac{x^n}{n} + \dots$$

$$e^x = \sum_{n=0}^{+\infty} \frac{x^n}{n!} \quad \forall x \in \mathbb{R}$$

rayon infini

II Calcul du rayon: $a_n = \frac{x^n}{n!}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{|x|^{n+1}}{(n+1)!} \frac{n!}{|x|^n} = \frac{|x|}{n+1} \xrightarrow{n \rightarrow +\infty} 0 < 1, \text{ donc } \sum a_n \text{ CV, } \forall x$$

$$\underline{R = +\infty}$$

$$S(x) = \sum_{n=0}^{+\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \dots \quad \text{On cherche.}$$

$$S'(x) = \sum_{n=1}^{+\infty} \frac{n x^{n-1}}{n!} = \sum_{n=1}^{+\infty} \frac{x^{n-1}}{(n-1)!} = \sum_{n=0}^{+\infty} \frac{x^n}{n!} = S(x)$$

$$S' - S = 0 \quad S = A e^{+x}$$

$$\text{valeur en 0: } S(x) = 1 + 0 + \frac{0}{2!} + \frac{0}{3!} + \dots = 1 = A \cdot e^0 = A \Rightarrow \underline{S(x) = e^x} \quad \square$$

$$\begin{aligned}
 e^t + e^{-t} &= 1 + \cancel{t} + \frac{t^2}{2!} + \frac{\cancel{t^3}}{3!} + \dots + \\
 &\quad + \left(1 - \cancel{t} + \frac{t^2}{2!} - \frac{\cancel{t^3}}{3!} + \dots + \right) \\
 &= 2 \left(1 + \frac{t^2}{2!} + \frac{t^4}{4!} + \dots \right)
 \end{aligned}$$

$$\cosh(t) = \sum_{n=0}^{+\infty} \frac{t^{2n}}{(2n)!} \quad \forall t \in \mathbb{R}$$

Rayon ∞

$$\begin{aligned}
 e^t - e^{-t} &= \cancel{1} + t + \frac{\cancel{t^2}}{2!} + \frac{t^3}{3!} + \frac{\cancel{t^4}}{4!} + \dots \\
 &\quad - \left(\cancel{1} - t + \frac{\cancel{t^2}}{2!} - \frac{t^3}{3!} + \frac{\cancel{t^4}}{4!} + \dots \right) = 2 \left(t + \frac{t^3}{3!} + \dots \right)
 \end{aligned}$$

$$\sinh(t) = \sum_{n=0}^{+\infty} \frac{t^{2n+1}}{(2n+1)!} \quad \forall t \in \mathbb{R}$$

Rayon ∞

$$e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \dots$$

$$= 1 + \cancel{ix} - \frac{x^2}{2!} - i \frac{x^3}{3!} + \frac{x^4}{4!} + i \frac{x^5}{5!} - \frac{x^6}{6!} + \dots$$

or replace x by $-x$

$$e^{-ix} = 1 - \cancel{ix} - \frac{x^2}{2!} + i \frac{x^3}{3!} + \frac{x^4}{4!} - i \frac{x^5}{5!} - \frac{x^6}{6!} + \dots$$

$$e^{ix} + e^{-ix} = 2 \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \right)$$

$$\cos x = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad \forall x \in \mathbb{R}$$

rayon ∞

$$\sin x = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad \forall x \in \mathbb{R}$$

rayon ∞

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots + \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!} x^n + \dots$$

$$= \sum_{n=0}^{+\infty} \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!} x^n$$

Beuk

Taylor : Si $f(x) = \sum_{n=0}^{+\infty} a_n x^n$ est une série entière de rayon $R > 0$,

$$\text{alors } a_n = \frac{f^{(n)}(0)}{n!}$$

$$f(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Il y a donc une seule façon
d'écrire un Dév^t en S. E.

$$\text{Si } \sum_{n=0}^{+\infty} a_n x^n = \sum_{n=0}^{+\infty} b_n x^n \text{ alors } a_n = b_n, \forall n$$

Il y a unicité
du Dév^t en S. E.