

llll

① def de la matrice de passage de la base \mathcal{A} à la base \mathcal{B}

$$P_{\mathcal{A} \rightarrow \mathcal{B}} = \begin{pmatrix} \text{coord} \\ \text{de } \vec{u} \\ \text{dans } \mathcal{A} & \vec{v} & \vec{w} \\ \text{dans } \mathcal{A} & \text{dans } \mathcal{A} & \text{dans } \mathcal{A} \end{pmatrix} \text{ où } \mathcal{B} = (\vec{u}, \vec{v}, \vec{w})$$

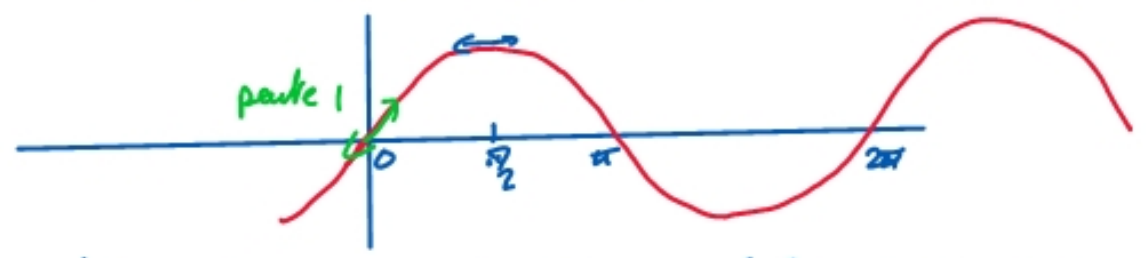
② Somme partielle géométrique

$$1 + x + x^2 + \dots + x^n = \frac{x^{n+1} - 1}{x - 1} \text{ qd } x \neq 1 \quad \text{et } = n+1 \text{ qd } x = 1$$

③ DL de $\frac{1}{1+x}$ à l'ordre n

$$\frac{1}{1+x} = 1 - x + x^2 - \dots + (-1)^n x^n + \mathcal{O}(x^{n+1})$$

④ Dessin de $\sin(x)$



⑤ Que vaut $\binom{7}{2}$?

$$\binom{7}{2} = \frac{7!}{2!(7-2)!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{6 \cdot 7}{2} = 3 \cdot 7 = 21$$

$$\begin{array}{r}
 f(x,y) = x^3 + y^3 + x^2y + xy^2 - 6x - 6y \\
 - (x^3 + x^2y) \\
 \hline
 xy^2 - 6x - 6y + y^3 \\
 - (xy^2 + y^3) \\
 \hline
 -6x - 6y \\
 - (-6x - 6y) \\
 \hline
 0
 \end{array}
 \quad \left| \begin{array}{r}
 x + y \\
 \hline
 x^2 + y^2 - 6
 \end{array} \right.$$

$$f(x,y) = (x+y)(x^2+y^2-6)$$

$$\begin{array}{r}
 x^2(\cancel{y^2-1}) - x(2\cancel{y^2-2}) + y^4 - y^2 + 1 \\
 \hline
 x^2(\cancel{y^2-1}) - 2x(\cancel{y^2-1}) + \underbrace{(y^2-1)^2}_{y^2}
 \end{array}$$

$$\begin{aligned}
 f(x,y) &= (x^2 + y^2 - 3)^2 + 4y^2 - 8 \quad \text{par } (x-1)^2 + y^2 - 2 = x^2 - 2x + y^2 - 1 \\
 &= x^4 + y^4 + 9 + 2x^2y^2 - 6x^2 - 6y^2 + 4y^2 - 8
 \end{aligned}$$

$$\begin{array}{r}
 = x^4 + x^2[2y^2 - 6] + y^4 - 2y^2 + 1 \\
 - (x^4 - 2x^3 + x^2(y^2 - 1)) \\
 \hline
 2x^3 + x^2(y^2 - 5) + y^4 - y^2 + 1 \\
 - (2x^3 - 4x^2 + 2xy^2 - 2x)
 \end{array}
 \quad \left| \begin{array}{r}
 x^2 - 2x + y^2 - 1 \\
 \hline
 x^2 + 2x + y^2 - 1
 \end{array} \right.$$

Bemerk

$$(8) \quad P(x) = x^4 - 2x^3 + 2x - 1$$

$$-(x^4 - x^3)$$

$$-x^3 + 2x - 1$$

$$-(-x^3 + x^2)$$

$$-x^2 + 2x - 1$$

$$-(-x^2 + x)$$

$$x - 1$$

$$-(x - 1)$$

$$0$$

$$x - 1$$

$$x^3 - x^2 - x + 1$$

$$-(x^3 - x^2)$$

$$-x + 1$$

$$-(-x + 1)$$

$$0$$

$$x - 1$$

$$x^2 - 1 = (x - 1)(x + 1)$$

$$-x^3 - 6x^2 - 11x - 6 \quad | \quad x + 2$$

$$-(x^3 + 2x^2)$$

$$-4x^2 - 11x - 6$$

$$-(-4x^2 - 8x)$$

$$-3x - 6$$

$$-(-3x - 6)$$

$$0$$

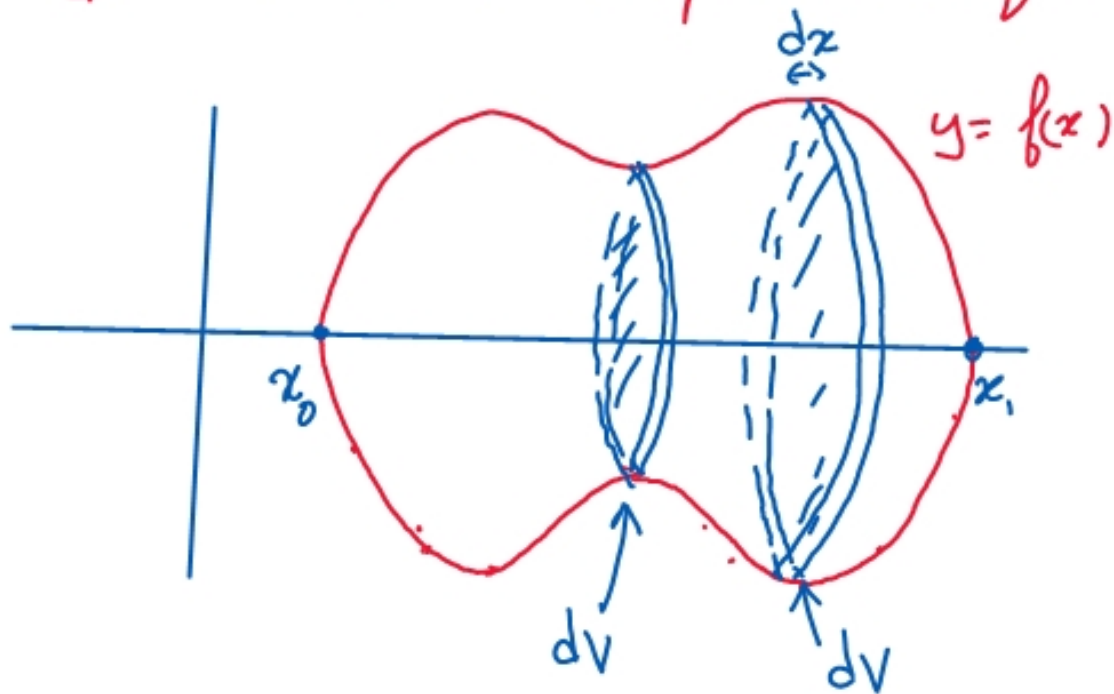
$$P(x) = (x - 1)(x - 1)(x - 1)(x + 1) = (x - 1)^3(x + 1)$$

$$(9) \quad P(x) = -x^3 + x^2 + 2x = x(-x^2 + x + 2) = x(x + 1)(-x + 2)$$

$$(10) \quad P(x) = -x^3 - 6x^2 - 11x - 6 = (x + 2)(-x^2 - 4x - 3) = (x + 2)(x + 1)(-x - 3)$$

$$P(-2) = +8 - 24 + 22 - 6 = 0$$

III Volumes délimités par une surface de révolution



$$dV = \pi y^2 dx$$

(aire du disque)

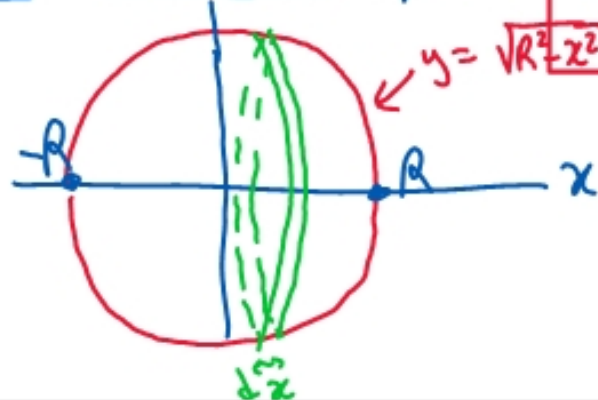
hauteur du cylindre

$$V = \int_{x_0}^{x_1} \pi y^2 dx$$

on apprend à le retrouver

ex: Volume de la sphère

$$V = \frac{4\pi R^3}{3}$$



$$dV = \pi y^2 dx$$

(aire du disque)

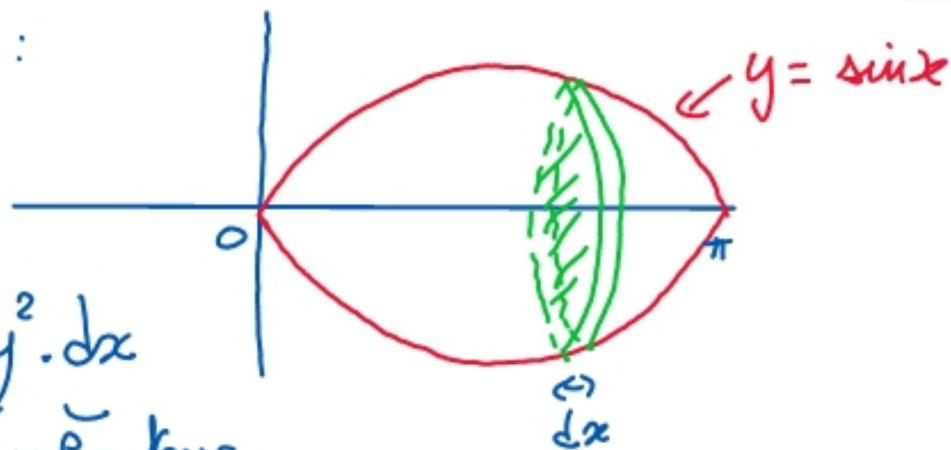
$$V = \int_{-R}^R = 2 \int_0^R \pi y^2 dx = 2 \int_0^R \pi (R^2 - x^2) dx$$

$$V = 2\pi \left[R^2 x - \frac{x^3}{3} \right]_0^R = 2\pi \left[R^3 - \frac{R^3}{3} \right] = 2\pi \left(\frac{3R^3 - R^3}{3} \right) = \frac{4\pi R^3}{3}$$

ex 2 : Volume du segmentaire :

Volume du cylindre élémentaire.

$$dV = \underbrace{\pi y^2}_{\text{base}} \cdot \underbrace{dx}_{\text{hauteur}}$$



$$V = \int_{x=0}^{x=\pi} \pi \sin^2 x \, dx = \pi \int_0^{\pi} \sin^2 x \, dx$$

Rappel : $\cos(2x) = 1 - 2\sin^2 x$

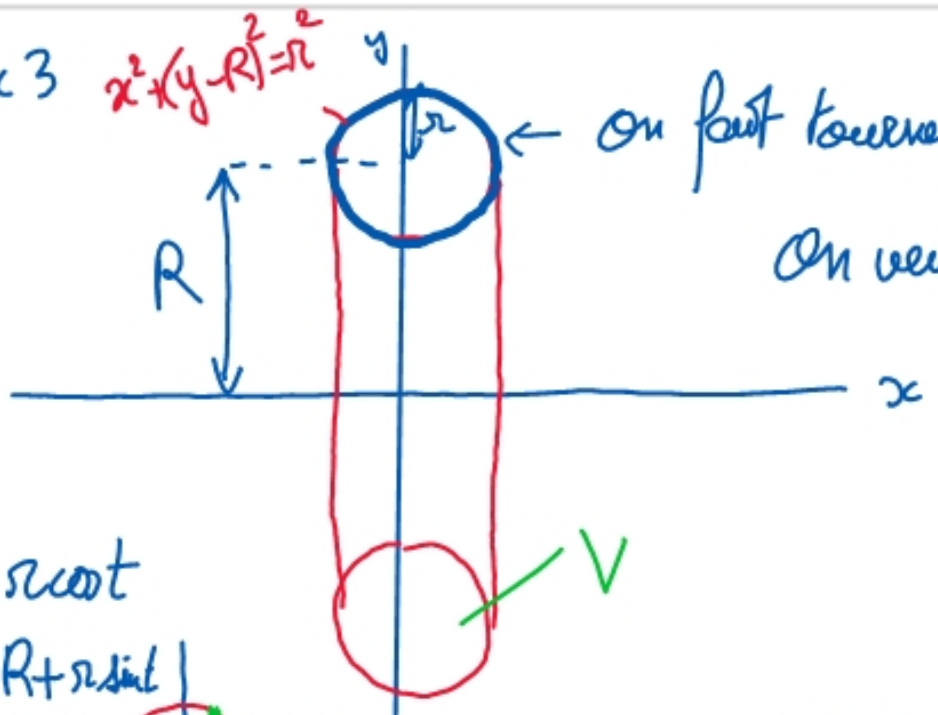
$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$V = \frac{\pi}{2} \int_0^{\pi} (1 - \cos(2x)) \, dx$$

$$V = \frac{\pi}{2} \left[x - \frac{\sin(2x)}{2} \right]_0^{\pi} = \frac{\pi}{2} \cdot \pi = \frac{\pi^2}{2}$$

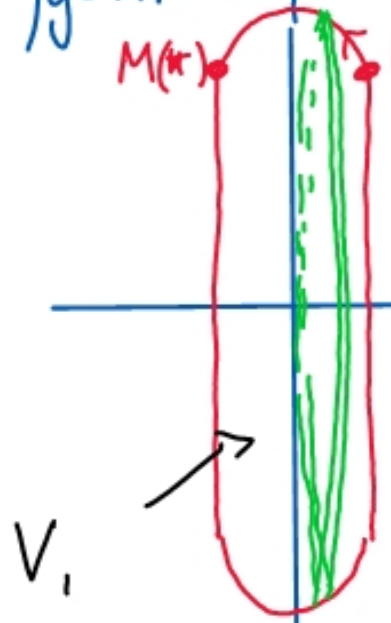
ex 3 $x^2 + (y-R)^2 = r^2$

on fait tourner ce cercle autour de l'axe des x
 on veut le volume des deux

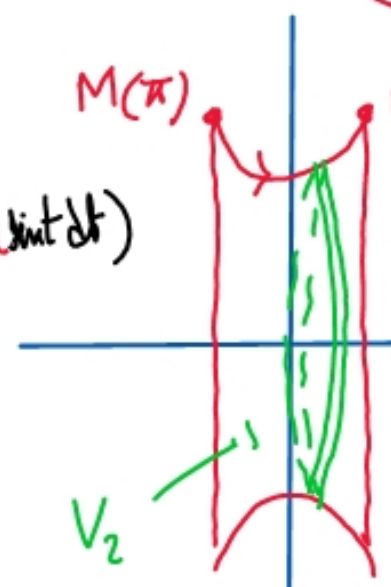


$$\begin{cases} x = r \cos t \\ y = R + r \sin t \end{cases}$$

$$V = V_1 - V_2$$



$$V_1 = \int_{t=0}^{t=\pi} \pi (R + r \sin t)^2 (-r \sin t) dt$$



$$V_2 = \int_{t=\pi}^{t=2\pi} \pi (R + r \sin t)^2 (-r \sin t) dt$$

$$V_1 = \int_{x=-r}^r \pi y^2 dx$$

$$V_2 = \int_{-r}^r \pi y_2^2 dx$$

$$V_1 - V_2 = \int_{-\pi}^0 - \int_{\pi}^{2\pi} = \int_{-\pi}^0 + \int_{\pi}^{2\pi} = \int_{-\pi}^0$$

$$= - \int_0^{\pi}$$

$$V = \int_0^{2\pi} \pi (R + r \sin t)^2 r \sin t dt$$

$$V = \pi r \int_0^{2\pi} (R + r \sin t)^2 \sin t dt$$

$$\begin{aligned} I &= \int (R + r \sin t)^2 \sin t dt = \int (R^2 + 2rR \sin t + r^2 \sin^2 t) \sin t dt \\ &= \int (R^2 \sin t + 2rR \sin^2 t + r^2 \sin^3 t) dt \end{aligned}$$

$$K = \int \sin t dt = -\cos t \quad L = \int \sin^2 t dt = \int \frac{1 - \cos(2t)}{2} dt = \frac{t}{2} - \frac{\sin(2t)}{4}$$

$$M = \int \sin^3 t dt \quad \text{chgt } X = \cos t \quad dX = -\sin t dt \Rightarrow dt = \frac{-dX}{\sin t}$$

$$M = \int \sin^3 t \frac{-dX}{\sin t} = \int -\sin^2 t dX = \int (\cos^2 t - 1) dX = \int (X^2 - 1) dX = \frac{X^3}{3} - X$$

$$M = \frac{\cos^3 t}{3} - \cos t$$

$$V = \pi r \left[-R^2 \cos t + \left(\frac{t}{2} - \frac{\sin(2t)}{4} \right) 2rR + r^2 \left(\frac{\cos^3 t}{3} - \cos t \right) \right]_0^{2\pi}$$

$$V = \pi r [\pi \cdot 2rR]$$

$$V = 2\pi^2 rR$$