

Wiederholung

dm 8 parte II

$$X^5 - 1 = (X-1) \left(X^2 + \frac{1+\sqrt{5}}{2} X + 1 \right) \left(X^2 + \frac{1-\sqrt{5}}{2} X + 1 \right) = (X-1) (X^4 + X^3 + X^2 + X + 1)$$

$$\frac{X^4 + 2X^3 + 2}{X^5 - 1} = E(x) + \frac{a}{X-1} + \frac{bx+c}{X^2 + \frac{1+\sqrt{5}}{2} X + 1} + \frac{dX+e}{X^2 + \frac{1-\sqrt{5}}{2} X + 1} = \frac{a}{X-1} + \frac{P(x)}{X^4 + X^3 + X^2 + X + 1}$$

$$\textcircled{1} \quad a = \left. \frac{X^4 + 2X^3 + 2}{X^4 + X^3 + X^2 + X + 1} \right|_{X=1} = \frac{5}{5} = 1$$

$$\textcircled{2} \quad \frac{X^4 + 2X^3 + 2}{(X-1)(X^4 + X^3 + X^2 + X + 1)} - \frac{1}{X-1} = \frac{X^4 + 2X^3 + 2 - (X^4 + X^3 + X^2 + X + 1)}{(X-1)(X^4 + X^3 + X^2 + X + 1)}$$

$$= \frac{+X^3 - X^2 - X + 1}{(X-1)(X^4 + X^3 + X^2 + X + 1)} = \frac{P(x)}{X^4 + X^3 + X^2 + X + 1}$$

$$\begin{array}{r|l} +X^3 - X^2 - X + 1 & X-1 \\ \hline -(X^3 - X^2) & X^2 - 1 \\ \hline -X + 1 & \\ -X + 1 & \\ \hline 0 & \end{array}$$

$$P(x) = X^2 - 1$$

$$(23) \quad k = \int \frac{1}{t^2 + t - 1} dt \quad \Delta = 5 \quad \text{raiz: } \frac{-1 \pm \sqrt{5}}{2}$$

$$k = \int \frac{1}{\left(t - \frac{-1+\sqrt{5}}{2}\right) \left(t - \frac{-1-\sqrt{5}}{2}\right)} dt$$

$$k = \int \left(\frac{a}{t - \frac{-1+\sqrt{5}}{2}} + \frac{b}{t - \frac{-1-\sqrt{5}}{2}} \right) dt$$

$$a = \frac{1}{t - \frac{-1-\sqrt{5}}{2}} \Bigg|_{t = \frac{-1+\sqrt{5}}{2}} = \frac{1}{\sqrt{5}}$$

$$b = \frac{1}{t - \frac{-1+\sqrt{5}}{2}} \Bigg|_{t = \frac{-1-\sqrt{5}}{2}} = -\frac{1}{\sqrt{5}}$$

$$k = \frac{1}{\sqrt{5}} \int \frac{1}{t - \frac{-1+\sqrt{5}}{2}} dt - \frac{1}{\sqrt{5}} \int \frac{1}{t - \frac{-1-\sqrt{5}}{2}} dt$$

$$= \frac{1}{\sqrt{5}} \ln \left| t - \frac{-1+\sqrt{5}}{2} \right| - \frac{1}{\sqrt{5}} \ln \left| t - \frac{-1-\sqrt{5}}{2} \right|$$

$$K = \int \frac{1}{t^2 + t - 1} dt$$

$$\text{chft } t = x + \frac{1}{x}$$

$$\frac{dt}{dx} = 1 - \frac{1}{x^2} \Rightarrow dt = \left(1 - \frac{1}{x^2}\right) dx$$

$$dt = \frac{x^2 - 1}{x^2} dx$$

$$K = \int \frac{1}{\left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) - 1} \frac{x^2 - 1}{x^2} dx$$

$$= \int \frac{1}{\left(x^2 + 2 + \frac{1}{x^2} + x + \frac{1}{x} - 1\right)} \frac{x^2 - 1}{x^2} dx$$

$$= \int \frac{x^2 - 1}{x^4 + x^3 + x^2 + x + 1} dx$$

Sols de l'eq homogène: dans \mathcal{B}' :

$$\begin{cases} x_1' = at + b \\ x_2' = \alpha \cos(\sqrt{3}t) + \beta \sin(\sqrt{3}t) \end{cases}$$

dans \mathcal{B} . $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underset{\mathcal{B} \rightarrow \mathcal{B}'}{\mathcal{P}} \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \leftarrow$ Formules de changement de base pour les vecteurs

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} at+b \\ \alpha \cos(\sqrt{3}t) + \beta \sin(\sqrt{3}t) \end{pmatrix}$$

$$x_1(t) = at + b + 2\alpha \cos(\sqrt{3}t) + 2\beta \sin(\sqrt{3}t)$$

$$x_2(t) = at + b - \alpha \cos(\sqrt{3}t) - \beta \sin(\sqrt{3}t)$$

↑
sols de HOM dans \mathcal{B}

Une solution particulière : $x_1(t) = 0$ et $x_2(t) = 1$ marchent

Les solutions de l'eq complète : $x_1(t) = at + b + 2\alpha \cos(\sqrt{3}t) + 2\beta \sin(\sqrt{3}t)$
 $x_2(t) = at + b - \alpha \cos(\sqrt{3}t) - \beta \sin(\sqrt{3}t) + 1$

La solution cherchée : $x_1(0) = 0 \Rightarrow \begin{cases} b + 2\alpha = 0 \\ b - \alpha + 1 = 1 \end{cases} \Rightarrow \begin{cases} 3\alpha = 0 \\ b = \alpha \end{cases} \Rightarrow \alpha = b = 0$

$$x_1(t) = at + 2\beta \sin(\sqrt{3}t) \Rightarrow \dot{x}_1(t) = a + 2\sqrt{3}\beta \cos(\sqrt{3}t)$$

$$x_2(t) = at - \beta \sin(\sqrt{3}t) + 1 \Rightarrow \dot{x}_2(t) = a - \beta\sqrt{3} \cos(\sqrt{3}t)$$

$$\dot{x}_1(0) = 3 = a + 2\sqrt{3}\beta$$

$$\dot{x}_2(0) = 0 = a - \beta\sqrt{3}$$

$$\Leftrightarrow a = \beta\sqrt{3} = 1$$

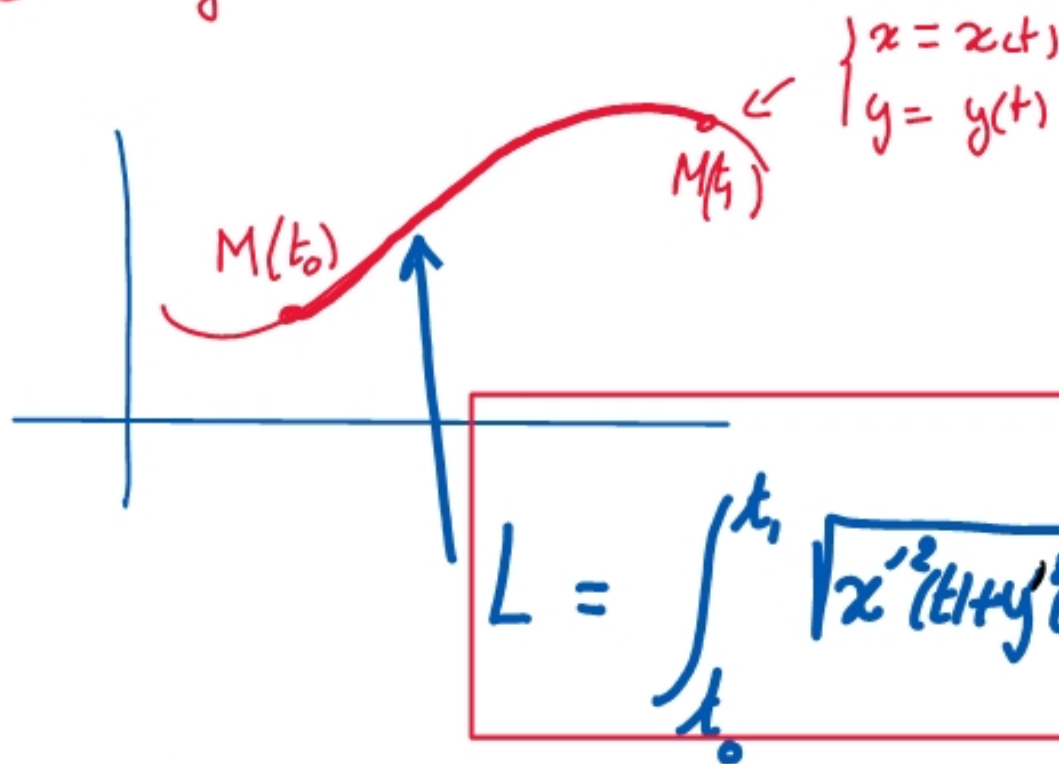
$$\begin{cases} a + 2\sqrt{3}\beta = 3 \\ a - \beta\sqrt{3} = 0 \end{cases} \Rightarrow \begin{matrix} \times 2 \\ - \\ \hline \end{matrix} \Rightarrow \begin{cases} 3\sqrt{3}\beta = 3 \\ \beta = \frac{1}{\sqrt{3}} \end{cases}$$

$$x_1(t) = t + \frac{2}{\sqrt{3}} \sin(\sqrt{3}t)$$

$$x_2(t) = t - \frac{1}{\sqrt{3}} \sin(\sqrt{3}t) + 1$$

Applications de calcul intégral

① longueur d'un arc



ex 1 longueur d'un arc de cercle

éq param d'un cercle

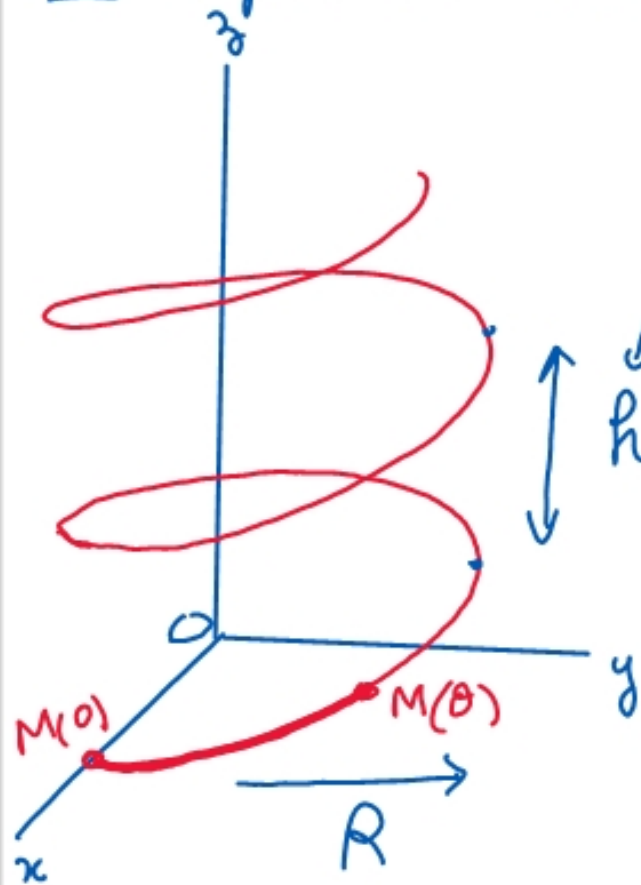
$$\begin{cases} x(t) = R \cos(t) \\ y(t) = R \sin(t) \end{cases}$$



$$L = \int_0^\theta \sqrt{R^2 \sin^2 t + R^2 \cos^2 t} dt = \int_0^\theta R \sqrt{1} dt = R [t]_0^\theta = R\theta$$

dans \mathbb{R}^3 :
$$L = \int_{t_0}^{t_1} \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} dt$$

ex2 : longueur d'un arc d'hélice circulaire :



pas de l'hélice
↓
h et π

$$\begin{cases} x(t) = R \cos t \\ y(t) = R \sin t \\ z(t) = h t \end{cases} \quad \begin{cases} x'(t) = -R \sin t \\ y'(t) = R \cos t \\ z'(t) = h \end{cases}$$

↑
pas

$$L = \int_0^{\theta} \sqrt{R^2 \sin^2 t + R^2 \cos^2 t + h^2} dt$$

$$= \int_0^{\theta} \sqrt{R^2 + h^2} dt = \sqrt{R^2 + h^2} [t]_0^{\theta}$$

$$L = \sqrt{R^2 + h^2} \theta$$

ex3 : Longueur de la sinusoïde de 0 à π



Ruse

$$\begin{aligned} x(t) &= t \\ y(t) &= \sin t \end{aligned}$$

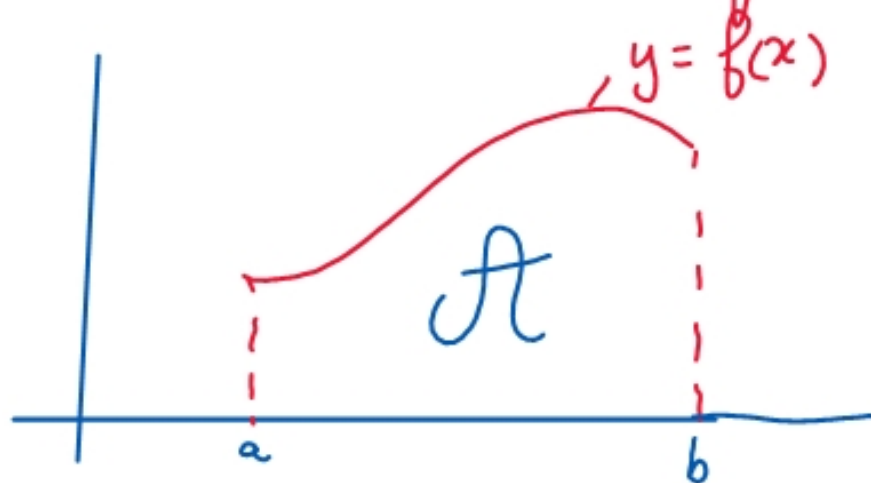
$$x'(t) = 1 \quad y'(t) = \cos(t)$$

$$L = \int_0^{\pi} \sqrt{1 + \cos^2(t)} dt$$

à voir

???

II Aire d'une surface plane



Si $f(x) \geq 0, \forall x \in [a, b]$, alors l'aire sous la courbe vaut

$$A = \int_a^b f(x) dx$$

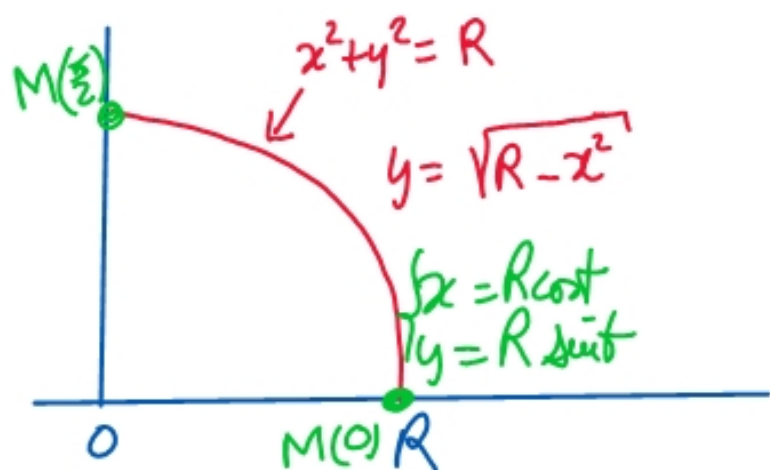
$$A = \int_a^b y dx$$

Si f change de signe $A = \int_a^b |y| dx$

ex 1 : Un

disque .

En cartésien



$$A = 4 \int_{x=0}^{x=R} \underbrace{\sqrt{R-x^2}}_y dx$$

En paramétré

$$A = 4 \int_{t=\frac{\pi}{2}}^{t=0} y dx$$

$$y = R \sin t$$

$$\frac{dx}{dt} = -R \sin t$$

⇓

$$dx = -R \sin t dt$$

$$A = 4 \int_{\frac{\pi}{2}}^0 R \sin t (-R \sin t dt)$$

$$A = -4 R^2 \int_{\frac{\pi}{2}}^0 \sin^2 t dt$$

$$A = 4 R^2 \int_0^{\frac{\pi}{2}} \sin^2 t dt$$

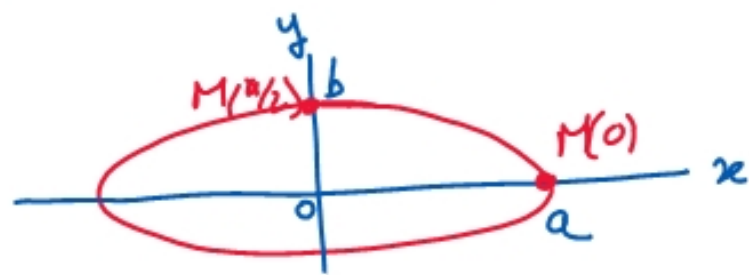
appel : $\cos^2 t = 1 - 2\sin^2 t$

$$A = 4R^2 \int_0^{\pi/2} \left(\frac{1}{2} - \frac{1}{2} \cos(2t) \right) dt$$

$$= 4R^2 \left[\frac{1}{2}t - \frac{1}{2} \frac{\sin(2t)}{2} \right]_0^{\pi/2} = 4R^2 \left[\underbrace{\frac{\pi}{4}}_{\text{en } \frac{\pi}{2}} - 0 - \underbrace{(0-0)}_{\text{en } 0} \right]$$

$$A = \pi R^2$$

ex2: aire d'une ellipse



eq paramétrique d'une ellipse

$$\begin{cases} x = a \cos t \\ y = b \sin t \end{cases} \Rightarrow dx = -a \sin(t) dt$$

$$A = 4 \int_{t=\pi/2}^{t=0} y dx = 4 \int_{\pi/2}^0 b \sin(t) \cdot (-a \sin(t) dt) = 4ab \int_0^{\pi/2} \sin^2(t) dt$$

$$A = \pi ab$$