

we

(1)

$$\text{Mat}_B(f) = \begin{pmatrix} 2 & -3 & -4 & 0 \\ 3 & 1 & 5 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 2 & 4 & 0 \end{pmatrix}$$

\uparrow \uparrow \uparrow \uparrow
 $f(\vec{e}_1)$ $f(\vec{e}_2)$ $f(\vec{e}_3)$ $f(\vec{e}_4)$

$$B = (\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4)$$

$$= \begin{pmatrix} 2 & 0 & 0 & 0 \\ 3 & 11 & 11 & 0 \\ -1 & -3 & -3 & 0 \\ 0 & 4 & 4 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 0 & 0 & 0 \\ 3 & 11 & 0 & 0 \\ -1 & -3 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

\uparrow \uparrow
 $f(\vec{e}_1 + 2f(\vec{e}_2))$
 $f(\vec{e}_1) + f(\vec{e}_2)$

$C_2 - C_3$
 $f(\vec{e}_1 + 2\vec{e}_2 - \vec{e}_3) = \vec{0}$

$$\text{Ker } f = \text{Vect} \left(\begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right)$$

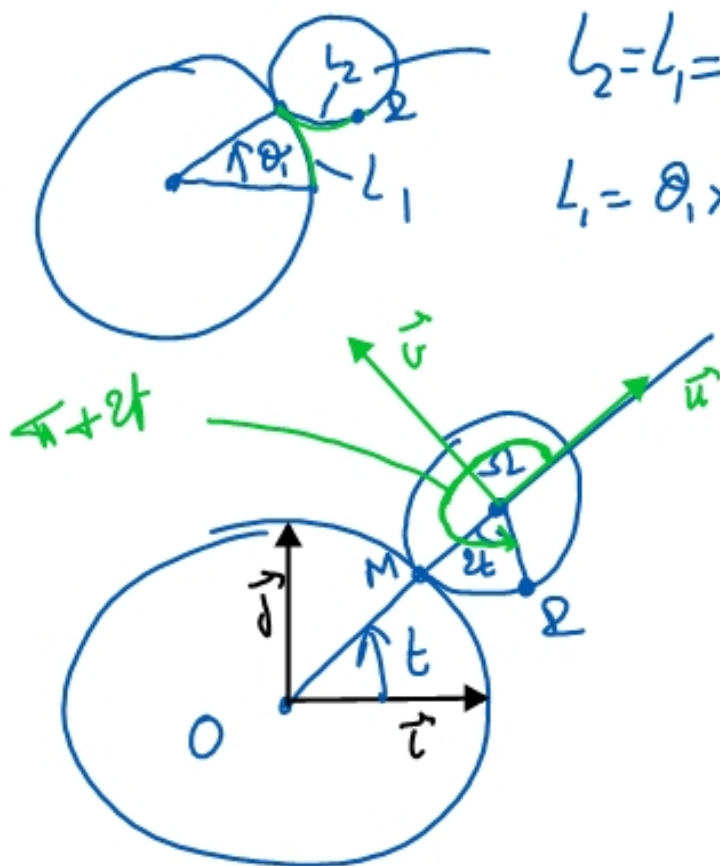
$$\text{Im } f = \text{Vect} \left(\begin{pmatrix} 2 \\ 3 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 11 \\ 3 \\ 4 \end{pmatrix} \right)$$

$$\text{rang } f = 2$$

①



longueur de l'arc: $L = \theta \times R$
 (tout le cercle: $L = 2\pi \times R$)



$$L_2 = L_1 = \theta_2 \times \frac{1}{2} \Rightarrow \theta_2 = 2\theta, \quad \mathcal{P} = (\Omega, \vec{u}, \vec{v})$$

$$L_1 = \theta_1 \times 1, \quad \mathcal{R} = (O, \vec{u}, \vec{v})$$

$$2) \vec{\Omega} = \theta \vec{u} + \theta \vec{v}$$

$$= 1 \vec{u} + \frac{1}{2} \vec{u} = \frac{3}{2} \vec{u}$$

$$\Omega = \left(\frac{3}{2} \omega ; \frac{3}{2} \dot{\omega} \right)$$

$$3) \vec{u} \cdot \vec{v} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \cdot \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} = 0 \text{ donc } \vec{u} \perp \vec{v}$$

$$\|\vec{u}\| = \|\vec{v}\| = \sqrt{\cos^2 t + \sin^2 t} = 1$$

$$4) \text{Coord de } P \text{ dans } \mathcal{P} : x_P = \frac{1}{2} (\cos(\pi + 2t); \sin(\pi + 2t)) = \left(-\frac{\cos(2t)}{2} ; -\frac{\sin(2t)}{2} \right)$$

(5) (x, y) et les coord de N dans \mathbb{R}^2 (\vec{i}, \vec{j})
 (X, Y) ————— \mathcal{P} (\vec{u}, \vec{v})

$$\vec{ON} = x\vec{i} + y\vec{j} = 0\vec{u} + \Omega\vec{v}$$

$$= \frac{3}{2}\cos t\vec{i} + \frac{3}{2}\sin t\vec{j} + X\vec{u} + Y\vec{v}$$

$$= \frac{3}{2}\cos t\vec{i} + \frac{3}{2}\sin t\vec{j} + X(\cos t\vec{i} + \sin t\vec{j}) + Y(-\sin t\vec{i} + \cos t\vec{j})$$

$$\vec{ON} = x\vec{i} + y\vec{j} = \left(\frac{3}{2}\cos t + X\cos t - Y\sin t\right)\vec{i} + \left(\frac{3}{2}\sin t + X\sin t + Y\cos t\right)\vec{j}$$

donc

$$x = \frac{3}{2}\cos t + X\cos t - Y\sin t$$

$$y = \frac{3}{2}\sin t + X\sin t + Y\cos t$$

$$\frac{3}{2} \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

matrice de rotation.

Techniques de Calcul Intégral

\pm Polynômes en sinus et cosinus

ex: On veut intégrer $P(x) = \sin^4 x + 3\cos^2 x \sin x - \cos^2 x \sin^2 x$

La linéarisation convient. Mais ...

On peut aller plus vite lorsqu'une puissance est impaire :

• $I = \int (\cos^p(x) \sin^q(x)) dx$ avec p impair
chgt de variable $X = \sin(x)$

• $I = \int (\cos^p(x) \sin^q(x)) dx$ avec q impair
chgt de variable $X = \cos(x)$

• $I = \int (\cos^p(x) \sin^q(x)) dx$ avec p et q pairs Sniff. Linéarisation

ex1: $I = \int \sin^4 x \, dx$ On linéaire

$$\sin^4 x = \left(\frac{e^{ix} - e^{-ix}}{2i} \right)^4 = \frac{1}{16} \left(\begin{array}{c} e^{i4x} - 4e^{i3x} \cdot e^{-ix} + 6e^{i2x} \cdot e^{-i2x} - 4e^{ix} \cdot e^{-i3x} + e^{-i4x} \end{array} \right)$$

$$= \frac{1}{16} \left(\frac{e^{i4x} + e^{-i4x}}{2 \cos(4x)} - 4 \frac{(e^{i2x} + e^{-i2x})}{2 \cos(2x)} + 6 \right) = \frac{1}{16} (\cos(4x) - 8 \cos(2x) + 6)$$

$$\sin^4 x = \frac{1}{8} \cos(4x) - \frac{1}{2} \cos(2x) + \frac{3}{8}$$

$$\int \sin^4 x \, dx = \frac{1}{32} \sin(4x) - \frac{1}{4} \sin(2x) + \frac{3}{8} x + \text{cte}$$

ex 2: $I = \int \cos^2 x \sin(x) dx$ chyt $X = \cos(x)$ $\frac{dX}{dx} = -\sin(x)$
 $\Rightarrow dx = -\frac{dX}{\sin(x)}$

$$I = \int X^2 \frac{\cancel{\sin(x)} dX}{-\cancel{\sin(x)}} = -\frac{X^3}{3} + cte = -\frac{\cos^3(x)}{3} + cte$$

ex 3. $I = \int \cos^2 x \sin^2 x dx$ On linéarise

$$\begin{aligned} \cos^2 x \sin^2 x &= \left(\frac{e^{ix} + e^{-ix}}{2} \right)^2 \left(\frac{e^{ix} - e^{-ix}}{2i} \right)^2 = -\frac{1}{16} (e^{i2x} + 2 + e^{-i2x}) (e^{i2x} - 2 + e^{-i2x}) \\ &= -\frac{1}{16} \left[e^{-i4x} - 2e^{i2x} + 1 + 2e^{i2x} - 4 + 2e^{-i2x} + 1 - 2e^{-i2x} + e^{-i4x} \right] \\ &= -\frac{1}{16} [2\cos(4x) - 2] = -\frac{1}{8}\cos(4x) + \frac{1}{8} \end{aligned}$$

$$\int \cos^2 x \sin^2 x dx = -\frac{1}{32} \sin(4x) + \frac{1}{8}x + cte$$

ex 4: $I = \int \sin^2 x \, dx$ On linéaire $\cos(2x) = 1 - 2 \sin^2(x)$

$$\Rightarrow \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$I = \int \left(\frac{1}{2} - \frac{1}{2} \cos(2x) \right) dx = \frac{1}{2} x - \frac{1}{4} \sin(2x) + \text{cte}$$

ex 5: $I = \int \cos^2(x) \sin^5(x) \, dx$ chgt $X = \cos(x)$ $\frac{dX}{dx} = -\sin(x)$

$$\Rightarrow dx = \frac{dX}{-\sin(x)}$$

$$I = \int X^2 \sin^5(x) \frac{dX}{-\sin(x)} = - \int X^2 \sin^4(x) dX$$

?? Rappel: $\cos^2(x) + \sin^2(x) = 1$

$$\Rightarrow \sin^2(x) = 1 - X^2$$

$$I = - \int X^2 (1 - X^2)^2 dX$$

$$= - \int X^2 (1 - 2X^2 + X^4) dX = \int (-X^6 + 2X^4 - X^2) dX = -\frac{X^7}{7} + 2\frac{X^5}{5} - \frac{X^3}{3} + \text{cte}$$

$$I = -\frac{\cos^7(x)}{7} + \frac{2}{5} \cos^5(x) - \frac{1}{3} \cos^3(x) + \text{cte}$$

II les fractions rationnelles

On réduit en éléments simples

- $\int \frac{1}{x} dx = \ln|x| + cste$

- $\int \frac{1}{x^2} dx = -\frac{1}{x} + cste$

car $\int x^n dx = \frac{x^{n+1}}{n+1} \quad n \neq -1$

(ex: $\int x^{-3} dx = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$
 $\int x^{-2} dx = \frac{x^{-1}}{-1} = -\frac{1}{x}$)

- $\int \frac{1}{x-1} dx = \ln|x-1| + cste$ et $\int \frac{1}{(x-1)^2} dx = -\frac{1}{x-1} + cste$

Cas général : $\int \frac{1}{(x-a)^\alpha} dx = \int (x-a)^{-\alpha} dx = \frac{(x-a)^{-\alpha+1}}{-\alpha+1} + \text{cte}$
qd $\alpha \neq 1$

• $\int \frac{1}{x^2+1} dx = \arctan(x)$

• $\int \frac{1}{x^2+4} dx = ?$ $\frac{1}{x^2+4} = \frac{1}{4} \left(\frac{1}{\frac{x^2}{4}+1} \right) = \frac{1}{4} \frac{1}{X^2+1}$

chgt $X = \frac{x}{2}$ avec $X = \frac{x}{2}$
 $\frac{dX}{dx} = \frac{1}{2} \Rightarrow dx = 2 dX$

$\int \frac{1}{x^2+4} dx = \frac{1}{4} \int \frac{1}{X^2+1} 2 dX = \frac{1}{2} \arctan(X) + \text{cte}$

$= \frac{1}{2} \arctan\left(\frac{x}{2}\right) + \text{cte}$

$$\bullet \int \frac{1}{x^2 - 2x + 5} dx = I \quad \Delta = 4 - 20 = -16$$

On se ramène à $\frac{1}{x^2 + 1}$

$$\underbrace{x^2 - 2x + 5}_{\text{}} = (x-1)^2 + 4 = 4 \left(\frac{(x-1)^2}{4} + 1 \right) = 4 \left(\left(\frac{x-1}{2} \right)^2 + 1 \right)$$

chgt $X = \frac{x-1}{2}$ $\frac{dx}{dX} = \frac{1}{2} \Rightarrow dx = 2 dX$

$$I = \int \frac{1}{4(x^2 + 1)} 2 dX = \frac{1}{2} \arctan(X) + cste = \underline{\underline{\frac{1}{2} \arctan\left(\frac{x-1}{2}\right) + cste}}$$

$$\bullet \int \frac{x}{x^2 + 1} dx = \frac{1}{2} \int \frac{2x}{x^2 + 1} dx = \frac{1}{2} \int \frac{u'}{u} = \frac{1}{2} \ln(x^2 + 1) + cste$$

$$\bullet \int \frac{2x+1}{x^2-6x+13} dx = I \quad \Delta < 0$$

$$\underline{x^2-6x+13} = (x-3)^2 + 4 = 4\left(\left(\frac{x-3}{2}\right)^2 + 1\right) \quad \text{chgt } X = \frac{x-3}{2}$$

$$\frac{dx}{2} = \frac{1}{2} \Rightarrow dx = 2dX$$

$$2X+3=x$$

$$I = \int \frac{2x+1}{4(x^2+1)} 2dX = \frac{1}{2} \int \frac{2(2X+3)+1}{X^2+1} dX = \frac{1}{2} \int \frac{4X+7}{X^2+1} dX$$

$$I = \frac{1}{2} \int \frac{4X}{X^2+1} dX + \frac{1}{2} \int \frac{7}{X^2+1} dX$$

$$= \ln(X^2+1) + \frac{7}{2} \arctan(X) + cte = \ln\left(\left(\frac{x-3}{2}\right)^2 + 1\right) + \frac{7}{2} \arctan\left(\frac{x-3}{2}\right) + cte$$