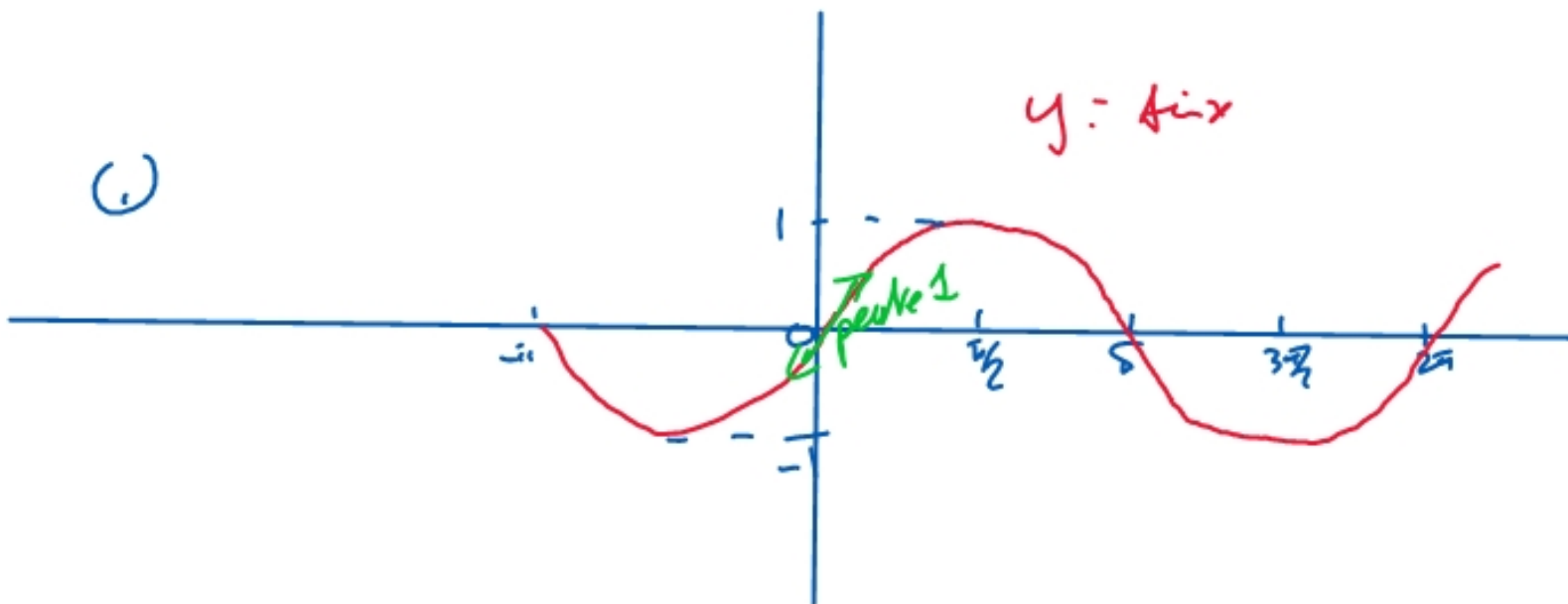


allern

(1)

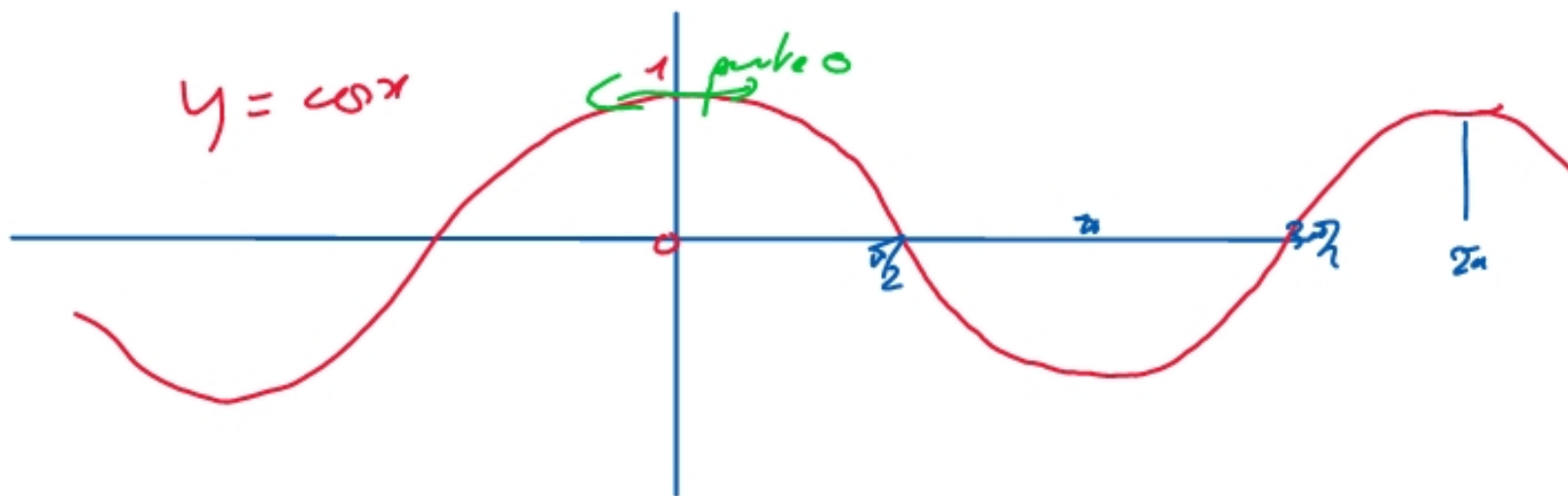


(2)

$$1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

$(\forall n \in \mathbb{N}^*)$

(3)



$$1 + x + x^2 + \dots + x^n = \begin{cases} \frac{x^{n+1} - 1}{x - 1} & \text{gol } x \neq 1 \\ n+1 & \text{gol } x = 1 \end{cases}$$

$$1 + 2 + 2^2 + \dots + 2^{10} = \frac{2^{11} - 1}{2 - 1} = \frac{2048 - 1}{1} = 2047$$

$$\textcircled{1} \sqrt{4^2} = 4 \quad \sqrt{3} \times \sqrt{3} = 2 \times 3 = 6 \quad -3\sqrt{2^2} = -3 \cdot 2 = -6$$

$$6\sqrt{2} \times \sqrt{3} = 6\sqrt{36} = 6 \cdot 6 = 36 \quad \sqrt{2^4 \times 3^6} = 2^2 \cdot 3^3 = 4 \cdot 27 = 108$$

$$\textcircled{2} 3\sqrt{2} = \sqrt{9 \cdot 2} = \sqrt{18} \quad 5\sqrt{3} = \sqrt{25 \cdot 3} = \sqrt{75}$$

$$3\sqrt{2} \sqrt{5} = \sqrt{9 \cdot 2 \cdot 5} = \sqrt{90}$$

$$\textcircled{3} \sqrt{45} + 2\sqrt{5} - 3\sqrt{20} = 3\sqrt{5} + 2\sqrt{5} - 3 \cdot 2\sqrt{5} = 5\sqrt{5} - 6\sqrt{5} = -\sqrt{5}$$

$$B = \sqrt{5} + 3\sqrt{500} = \sqrt{5} + 30\sqrt{5} = 31\sqrt{5}$$

$$C = (5\sqrt{2} + \sqrt{3})^2 = 25 \cdot 2 + 10\sqrt{6} + 3 = 53 + 10\sqrt{6}$$

$$D = \left(\frac{\sqrt{2}}{3} - 1\right)^2 = \left(\frac{\sqrt{2} - 3}{3}\right)^2 = \frac{2 - 6\sqrt{2} + 9}{9} = \frac{11 - 6\sqrt{2}}{9}$$

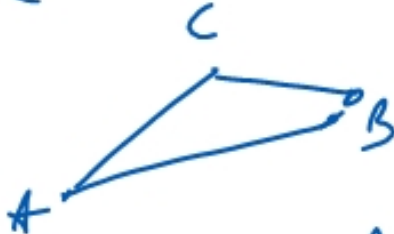
$$E = \frac{3\sqrt{5} + \sqrt{20}}{\left(2 - \frac{5}{6} + \frac{4}{3}\right)\sqrt{5}} = \frac{3\sqrt{5} + 2\sqrt{5}}{\left(\frac{12 - 5 + 8}{6}\right)3\sqrt{5}} = \frac{5\sqrt{5}}{\frac{15}{6} \cdot 3\sqrt{5}} = \frac{5}{3} \times \frac{6}{15}$$

$$E = \frac{\cancel{8} \times 2 \times \cancel{3}}{3 \times \cancel{3} \times \cancel{8}} = \frac{2}{3}$$

$$(4) \quad \frac{1}{4}a \times \frac{2a}{3\sqrt{2}} = \frac{2a^2}{12\sqrt{2}} = \frac{\cancel{2}a^2 \times \sqrt{2}}{6 \times \cancel{2}\sqrt{2} \times \sqrt{2}} = \frac{a^2\sqrt{2}}{12}$$

$$B = (a\sqrt{6})^2 - (2\sqrt{a})^2 = 6a^2 - 16a^2 = -10a^2$$

(5)



$$AB^2 = (\sqrt{3} + 2\sqrt{2})^2 = 3 + 8 + 4\sqrt{6} = 11 + 4\sqrt{6} = a$$

$$AC^2 = 18 = b$$

$$BC^2 = (4 + \sqrt{6})^2 = 4 + 4\sqrt{6} + b = 10 + 4\sqrt{6} = c$$

$$a + b \neq c \quad a + c \neq b \quad b + c \neq a$$

donc le triangle n'est pas rectangle

$$(6) \quad \frac{2\sqrt{3}}{\sqrt{2}} = \frac{2\sqrt{3}\sqrt{2}}{2} \sqrt{6} \quad ; \quad B = \frac{1}{1+\sqrt{2}} = \frac{1-\sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{1-\sqrt{2}}{1^2-2} = \frac{1-\sqrt{2}}{-1} = \sqrt{2}-1$$

$$C = \frac{5+2\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{5\sqrt{3}+5\sqrt{2}+2\sqrt{6}+4}{3-2} = 5\sqrt{3}+5\sqrt{2}+2\sqrt{6}+4$$

# VI Principes de calculs

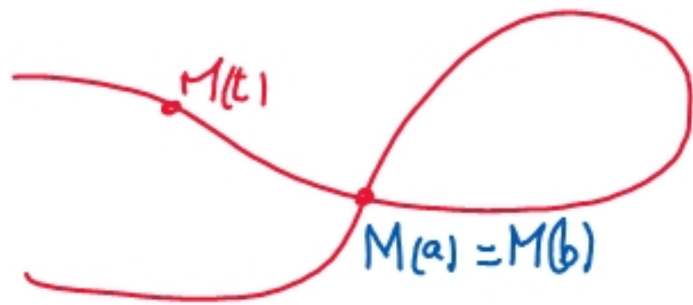
## 6.1. Netteyer

$$\text{ex: } \frac{2}{5} + \frac{3}{7} = \frac{14+15}{35} = \frac{29}{35}$$

$$\frac{2}{5} + 1 = \frac{2+5}{5} = \frac{7}{5}$$

## 6.2 Qui se ressemble s'assemble.

calcul des point double de l'arc paramétré

$$\begin{cases} x(t) = \frac{t^3}{t^2-9} \\ y(t) = \frac{t(t-2)}{t-3} \end{cases}$$


on voit qu'il existe  $a \neq b$

tel que  $M(a) = M(b)$ .

Coordonnées de M?

$$\begin{cases} \frac{a^3}{a^2-9} = \frac{b^3}{b^2-9} \\ \frac{a(a-2)}{a-3} = \frac{b(b-2)}{b-3} \end{cases} \Leftrightarrow \begin{cases} a^3(b^2-9) = b^3(a^2-9) \\ (a^2-2a)(b-3) = (b^2-2b)(a-3) \end{cases}$$

$$\Leftrightarrow \begin{cases} a^3b^2 - 9a^3 = b^3a^2 - 9b^3 \\ a^2b - 3a^2 - 2ab + 6a = ab^2 - 2ab - 3b^2 + 6b \end{cases}$$

on met ensemble ceux qui se rassemblent

$$\Leftrightarrow \begin{cases} a^3b^2 - b^3a^2 = 9a^3 - 9b^3 \\ (a^2b - ab^2) + 6(a-b) = 3(a^2 - b^2) \end{cases}$$

$$\begin{cases} a^2b^2 \cancel{(a-b)} = 9(a^3 - b^3) = 9\cancel{(a-b)}(a^2 + ab + b^2) \\ ab\cancel{(a-b)} + 6\cancel{(a-b)} = 3(a+b)\cancel{(a-b)} \end{cases}$$

$$\begin{array}{l} a^3 - b^3 \\ \hline (a^3 - a^2b) \\ a^2b - b^3 \\ \hline (a^2b - ab^2) \\ ab^2 - b^3 \\ \hline \end{array} \left| \begin{array}{l} a-b \\ \hline a^2 + ab + b^2 \end{array} \right.$$

$$\begin{cases} a^2 b^2 = 9(a^2 + ab + b^2) \\ ab + 6 = 3(a + b) \end{cases}$$

Ruse: On utilise  $P = ab$   
et  $S = a + b$

$$\begin{aligned} a^2 + ab + b^2 &= a^2 + 2ab + b^2 - ab \\ &= (a + b)^2 - ab = S^2 - P \end{aligned}$$

$$\textcircled{1} \quad P^2 = 9(S^2 - P)$$

$$\textcircled{2} \quad P + 6 = 3S \quad \Rightarrow \quad P = 3S - 6$$

$$\textcircled{1} \quad (3S - 6)^2 = 9(S^2 - 3S + 6)$$

$$\cancel{9S^2} - 36S + 36 = \cancel{9S^2} - 27S + 54$$

$$-9S = \underline{54 - 36} = 18 \quad \Rightarrow \quad \boxed{S = -2}$$

$$P = 3S - 6 = 3(-2) - 6 = -12$$

$$\boxed{P = -12}$$

$$\begin{cases} a+b = -2 \\ ab = -12 \end{cases} \Leftrightarrow \begin{cases} b = -2-a \\ a(-2-a) = -12 \Rightarrow -2a - a^2 = -12 \end{cases}$$

$$a^2 + 2a - 12 = 0 \quad \Delta = 4 + 48 = 52 = 4 \cdot 13$$

on trouve enfin a et b !!  $\sqrt{\Delta} = 2\sqrt{13}$

$$a = \frac{-2 \pm 2\sqrt{13}}{2} = -1 \pm \sqrt{13}$$

$$b = -2 - a = -2 + 1 \pm \sqrt{13} = -1 \pm \sqrt{13}$$

les deux valeurs possibles de a et b sont  $-1 \pm \sqrt{13}$ .

$$x(a) = \frac{a^3}{a^2 - 9} = \frac{(\sqrt{13}-1)^3}{(\sqrt{13}-1)^2 - 9} \quad \left( \begin{array}{l} a = \sqrt{13} - 1 \\ b = -\sqrt{13} - 1 \end{array} \right.$$

$$x(a) = \frac{13\sqrt{13} - 3 \cdot 13 + 3 \cdot \sqrt{13} - 1}{13 - 2\sqrt{13} + 1 - 9} = \frac{16\sqrt{13} - 40}{5 - 2\sqrt{13}} = \frac{8(2\sqrt{13} - 5)}{5 - 2\sqrt{13}} = \boxed{-8}$$



on trouve  $y(a) = -4$

Le point double a pour coordonnées  $(-8, -4)$

### 6.3 Identités remarquables

$$(a+b)^2 = a^2 + 2ab + b^2 \quad (a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$$

ex:  $a^3 + 3a^2 + 3a + 1 = (a+1)^3$

## 6.4 ATTENTION

• Aux racines  $\sqrt{a^2} = |a| = \begin{cases} a & \text{qd } a \geq 0 \\ -a & \text{qd } a \leq 0 \end{cases}$

ex:  $|x-1| = \begin{cases} x-1 & \text{qd } x \geq 1 \\ 1-x & \text{qd } x \leq 1 \end{cases}$

• ceux traits de fraction On ne divise pas par 0

• ceux simplifications On ne simplifie pas par 0

• ceux inégalités

$$a \leq b \Rightarrow \cancel{a^2 \leq b^2}$$
$$-3 \leq 2 \quad 9 > 4$$

$$\cancel{\frac{a}{b} \leq \frac{c}{d} \Rightarrow ad \leq bc}$$

RUSE:

Pour résoudre  $a \leq b$ , on résout  $a = b$ , puis on fait un tableau de signe de  $a - b$

Ex: trouver les  $x$  tels que  $x^2 < 3x - 2$

$$x^2 = 3x - 2 \Leftrightarrow x^2 - 3x + 2 = 0 \quad \Delta = 9 - 8 = 1$$

$$x = \frac{3 \pm 1}{2} = \begin{matrix} 1 \\ 2 \end{matrix}$$

$x$	$-\infty$	1	2	$+\infty$	
$x^2 - 3x + 2$	+	0	-	0	+
	$x^2 > 3x - 2$	$x^2 < 3x - 2$	$x^2 > 3x - 2$		

$$\underline{S = ]1; 2[}$$

