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①  $\sqrt{27} = \sqrt{3 \cdot 9} = \sqrt{3} \cdot \sqrt{9} = \underline{3\sqrt{3}}$

$\sqrt{200} = \sqrt{2 \cdot 100} = 10\sqrt{2}$  cau  $100 = 10^2$

$(-3\sqrt{2})^2 = 9 \cdot 2 = \underline{18}$

$\sqrt{a^2} = |a|$    
 $\sqrt{(-3)^2} = 3$

$\sqrt{8} \cdot 3\sqrt{2} \cdot 4\sqrt{3} = 3\sqrt{16} \cdot 4\sqrt{3} = 3 \cdot 4 \cdot 4 \cdot \sqrt{3} = \underline{48\sqrt{3}}$

$\sqrt{5} \times 2 \times \sqrt{45} = \sqrt{5} \times 2 \times \sqrt{5 \times 9} = \sqrt{5} \cdot \sqrt{5} \cdot 2 \cdot 3 = 5 \cdot 2 \cdot 3 = \underline{30}$

②  $2\sqrt{3} = \sqrt{4} \cdot \sqrt{3} = \sqrt{12}$      $3\sqrt{5} = \sqrt{9} \cdot \sqrt{5} = \sqrt{45}$      $2\sqrt{2} = \sqrt{4} \cdot \sqrt{2} = \sqrt{8}$

$4\sqrt{2} \cdot 2\sqrt{5} = \sqrt{16} \sqrt{2} \sqrt{4} \sqrt{5} = \sqrt{64 \cdot 10} = \sqrt{640}$

③  $2\sqrt{3} - 5\sqrt{3} + \sqrt{3} = -2\sqrt{3}$

$A = \sqrt{3}(2 - 5 + 1) = \sqrt{3}(-2) = -2\sqrt{3}$

$B = (2\sqrt{5})^2 - \left(\frac{5}{\sqrt{5}}\right)^2 = 4 \cdot 5 - 1 = 19$

$C = (\underbrace{\sqrt{2}} + \underbrace{\sqrt{5}}) \cdot (3\sqrt{2} - 2\sqrt{5}) = \underline{3 \cdot 2} - \underline{2 \cdot 10} + \underline{3\sqrt{10}} - \underline{2 \cdot 5} = 6 + \sqrt{10} - 10 = \underline{\sqrt{10} - 4}$

$$D = (\sqrt{27} - \sqrt{8})(2\sqrt{12} + \sqrt{50})$$

$$= (\underline{3\sqrt{3}} - \underline{2\sqrt{2}})(4\sqrt{3} + 5\sqrt{2}) = \underline{12 \cdot 3} + \underline{15\sqrt{6}} - \underline{8\sqrt{6}} - \underline{10 \cdot 2}$$

$$= 36 + 7\sqrt{6} - 20 = 16 + 7\sqrt{6}$$

$$E = \frac{2\sqrt{10}}{\sqrt{5}} \times \frac{3\sqrt{2}}{\sqrt{25}} = \frac{2\sqrt{2}\sqrt{5}}{\sqrt{5}} \times \frac{3\sqrt{2}}{5} = \frac{6 \cdot 2}{5} = \frac{12}{5}$$

$$\textcircled{4} \quad a\sqrt{3} - a\frac{\sqrt{3}}{3} = \frac{3a\sqrt{3} - a\sqrt{3}}{3} = \frac{2a\sqrt{3}}{3} \quad \left( = \frac{2a}{\sqrt{3}} \right)$$

$$B = (a\sqrt{5})^2 + \left(\frac{a}{2}\right)^2 = 5a^2 + \frac{a^2}{4} = \frac{5a^2 \times 4 + a^2}{4} = \frac{21a^2}{4}$$

$$\textcircled{5} \quad A = \frac{2}{\sqrt{5}} = \frac{2 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$B = \frac{7\sqrt{5}}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{15}}{12}$$

$$(a+b)(a-b) = a^2 - b^2$$

$$C = \frac{\sqrt{3}}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})} = \frac{\sqrt{3}+3}{1^2-3} = \frac{3+\sqrt{3}}{-2} = -\frac{3+\sqrt{3}}{2}$$

$$D = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{(2+\sqrt{3})} \cdot \frac{(2-\sqrt{3})}{(2-\sqrt{3})} + \frac{1}{(2-\sqrt{3})} \cdot \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$

$$= \frac{6\sqrt{3}}{3} + \frac{2-\sqrt{3}}{2^2-3} + \frac{2+\sqrt{3}}{2^2-3} \quad 2^2-3=1$$

$$= 2\sqrt{3} + \cancel{2-\sqrt{3}} + \cancel{2+\sqrt{3}} = 4 + 2\sqrt{3}$$

$$\boxed{i^2 = -1}$$

$$(a+ib)(a-ib) = a^2 + b^2$$

on dit que  $a-ib$  est le conjugué de  $a+ib$ .

Formule des binômes de Newton. Cas général

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n-1} a b^{n-1} + \binom{n}{n} b^n$$

$\uparrow$                      $\uparrow$                      $\uparrow$                      $\uparrow$                      $\uparrow$   
 1                    n                     $\frac{n(n-1)}{2}$                     n                    1

$$\binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{\cancel{2} \cdot \cancel{3} \dots (n-2)(n-1)n}{2 \cdot \cancel{2} \cdot \cancel{3} \dots (n-2)} = \frac{(n-1)n}{2}$$

$$(a+b)^n = \sum_{k=0}^{k=n} \binom{n}{k} a^{n-k} b^k = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$$

## Linéarisation:

Un complexe s'écrit  $a+ib$  ou  $re^{i\theta}$   
forme algébrique ou cartésienne

forme trigo ou exponentielle

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$



$$(e^{i\theta})^n = e^{in\theta}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$



Linéarisation de  $\cos^3 x$

$$\cos^3 x = \left( \frac{e^{ix} + e^{-ix}}{2} \right)^3 = \frac{1}{8} (e^{ix} + e^{-ix})^3 \quad \text{car } 2^3 = 8$$

$$= \frac{1}{8} \left( (e^{ix})^3 + 3(e^{ix})^2 e^{-ix} + 3e^{ix} (e^{-ix})^2 + (e^{-ix})^3 \right)$$

$$\left( (a+b)^3 = a^3 + 3a^2 b + 3a b^2 + b^3 \right)$$

$$= \frac{1}{8} \left( e^{i3x} + 3e^{i2x} \cdot e^{-ix} + 3e^{ix} e^{-i2x} + e^{-i3x} \right)$$

$$= \frac{1}{8} \left( e^{i3x} + 3e^{ix} + 3e^{-ix} + e^{-i3x} \right)$$

$$= \frac{1}{8} \left( \underbrace{3(e^{ix} + e^{-ix})} + \underbrace{(e^{i3x} + e^{-i3x})} \right)$$

$$= \frac{1}{8} \left( 3 \cdot 2\cos(x) + 2\cos(3x) \right)$$

$$\cos^3(x) = \frac{3}{4} \cos(x) + \frac{1}{4} \cos(3x)$$

Linearisation of  $\sin^4(x)$

$$i^2 = -1 \Rightarrow i^4 = (-1)^2 = 1$$

$$\sin^4(x) = \left( \frac{e^{ix} - e^{-ix}}{2i} \right)^4 = \frac{1}{16} \left( e^{i4x} - 4e^{i3x} \cdot e^{-ix} + 6e^{i2x} \cdot e^{-i2x} - 4e^{ix} \cdot e^{-i3x} + e^{-i4x} \right)$$

$$= \frac{1}{16} \left( e^{i4x} - 4e^{i2x} + \underbrace{6e^0}_1 - 4e^{-i2x} + e^{-i4x} \right)$$

$$= \frac{1}{16} \left( (e^{i4x} + e^{-i4x}) - 4(e^{i2x} + e^{-i2x}) + 6 \right)$$

$$= \frac{1}{16} \left( 2\cos(4x) - 4(2\cos(2x)) + 6 \right) = \underline{\underline{\frac{1}{8}\cos(4x) - \frac{1}{2}\cos(2x) + \frac{3}{8}}}$$

$$\cos^2(3x) \cdot \sin^3(x) = \left( \frac{e^{i3x} + e^{-i3x}}{2} \right)^2 \left( \frac{e^{ix} - e^{-ix}}{2i} \right)^3 = a$$

$$a = \frac{1}{4} \cdot \left( -\frac{1}{8i} \right) \left( e^{i6x} + 2e^{i3x}e^{-i3x} + e^{-i6x} \right) \left( e^{i3x} - 3e^{i2x}e^{-ix} + 3e^{ix}e^{-i2x} - e^{-i3x} \right)$$

$$= -\frac{1}{32i} \left( \underbrace{e^{i6x}} + \underbrace{2} + \underbrace{e^{-i6x}} \right) \left( e^{i3x} - 3e^{ix} + 3e^{-ix} - e^{-i3x} \right)$$

$$= -\frac{1}{32i} \left[ \begin{array}{l} e^{i9x} - 3e^{i7x} + 3e^{i5x} - e^{i3x} \\ + 2e^{i3x} - 6e^{ix} + 6e^{-ix} - 2e^{-i3x} \\ + e^{-i3x} - 3e^{-i5x} + 3e^{-i7x} - e^{-i9x} \end{array} \right]$$

$$= -\frac{1}{32i} \left[ \left( e^{i9x} - e^{-i9x} \right) - 3 \left( e^{i7x} - e^{-i7x} \right) + 3 \left( e^{i5x} - e^{-i5x} \right) + \left( e^{i3x} - e^{-i3x} \right) - 6 \left( e^{ix} - e^{-ix} \right) \right]$$



$$= -\frac{1}{32i} \left[ 2i \sin(9x) - 6i \sin(7x) + 6i \sin(5x) + 2i \sin(3x) - 12i \sin(x) \right]$$

$$= -\frac{1}{16} \sin(9x) + \frac{3}{16} \sin(7x) - \frac{3}{16} \sin(5x) - \frac{1}{16} \sin(3x) + \frac{3}{8} \sin(x)$$