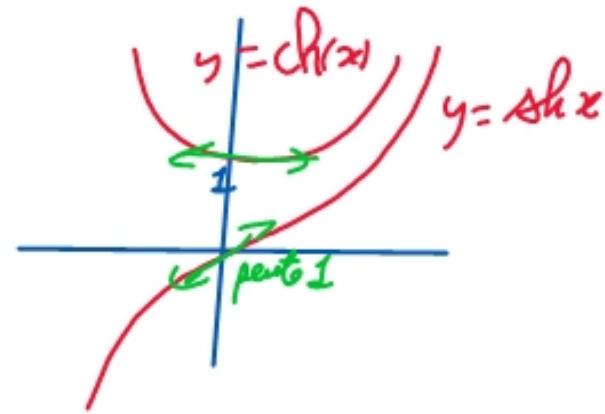


① ch et sh sur un même dessin



② dérivées de $\tan(x)$

$$\tan' x = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

③ $\cos(2x) = ?$ $= ?$

$$\cos(2x) = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

④ DL de $\cos x$ à l'ordre 3

$$\cos x = 1 - \frac{x^2}{2} + o(x^3)$$

⑤ formule des binômes

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

$$\textcircled{1} \quad I = \int \frac{\sin x}{1 + \cos^2 x} dx$$

$$\text{chgt } X = \cos x \quad \frac{dX}{dx} = -\sin x$$

$$\Rightarrow dx = -\frac{dX}{\sin x}$$

$$I = \int \frac{\cancel{\sin x}}{1 + X^2} \frac{-dX}{\cancel{\sin x}} = -\int \frac{1}{1 + X^2} dX = -\arctan(X) + \text{cte}$$

$$= -\arctan(\cos x) + \text{cte}$$

$$\textcircled{5} \quad I = \int \frac{1}{x \ln x \ln(\ln x)} dx$$

$$\text{chgt } X = \ln x \quad \frac{dX}{dx} = \frac{1}{x}$$

$$\Rightarrow dx = x dX$$

$$I = \int \frac{1}{x \times \ln(x)} x dX$$

$$\text{chgt } t = \ln(X)$$

$$\frac{dt}{dX} = \frac{1}{X} \Rightarrow dX = X dt$$

$$I = \int \frac{1}{X t} X dt = \ln(t)$$

$$= \ln(\ln(X)) = \underline{\ln(\ln(\ln x))}$$

$$\textcircled{10} \quad I = \int \frac{x^2}{\cos^2(x^3)} dx \quad X = x^3 \quad \frac{dX}{dx} = 3x^2 \\ \Rightarrow dx = \frac{dX}{3x^2}$$

$$I = \int \frac{\cancel{x^2}}{\cos^2(X)} \frac{dX}{3\cancel{x^2}} = \frac{1}{3} \int \frac{1}{\cos^2(X)} dX$$

$$= \frac{1}{3} \tan(X) + \text{cte} = \frac{1}{3} \tan(x^3) + \text{cte}$$

$$\textcircled{9} \quad I = \int_1^e \frac{1}{x(1+\ln^2 x)} dx \quad X = \ln(x) \quad \frac{dX}{dx} = \frac{1}{x} \Rightarrow dx = x dX$$

$$I = \int_0^1 \frac{1}{x(1+x^2)} x dx = [\arctan X]_0^1 = \arctan(1) - \arctan 0 \\ = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

$$\textcircled{7} \quad I = \int_0^{\frac{\pi}{2}} \cos^4 x \sin x \, dx$$

$$\text{choix } X = \cos x$$

$$\frac{dX}{dx} = -\sin x \Rightarrow dx = -\frac{dx}{\sin x}$$

$$I = \int_1^0 X^4 \sin x \frac{-dx}{\sin x} = - \int_1^0 X^4 dX = - \left[\frac{X^5}{5} \right]_1^0$$

$$= - \left(0 - \frac{1}{5} \right) = \frac{1}{5}$$

III Les endomorphismes de \mathbb{R}^n

Rappel: Un endomorphisme de \mathbb{R}^n est une application linéaire qui va de \mathbb{R}^n dans \mathbb{R}^n

T de rang: $\dim(\text{Ker}(f)) + \dim(\text{Im}(f)) = \dim(\text{Espace de départ})$

pour les endomorphismes de \mathbb{R}^n , l'ensemble de départ est aussi l'ensemble d'arrivée. $\forall x \in \mathbb{R}^n$

Si f est injective, alors $\text{Ker}(f) = \{ \vec{0} \} \Rightarrow \dim(\text{Ker}(f)) = 0$

$\Rightarrow \dim(\text{Im}(f)) = n$
 $\left. \begin{array}{l} \text{Im}(f) \text{ est un sev de } \mathbb{R}^n \\ \dim(\text{Im}(f)) = n \end{array} \right\} \text{ donc } \text{Im}(f) = \mathbb{R}^n \Rightarrow \text{f est } \underline{\text{surjective}}$

II Si f est un endomorphisme alors $f \text{ est injective} \Leftrightarrow f \text{ est surjective} \Leftrightarrow f \text{ est bijective}$

$$f(x) = \frac{x^2+1}{x} \quad (1) \quad D_f = \mathbb{R}^*$$

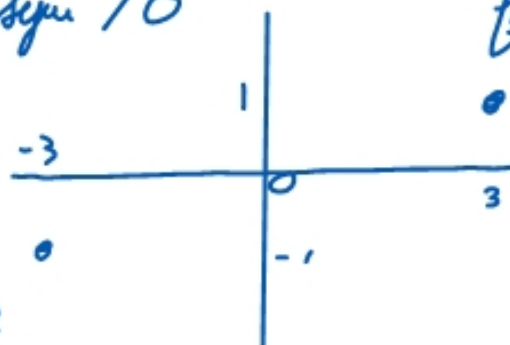
(2) pas de période

$$f(-x) = \frac{x^2+1}{-x} = -\frac{x^2+1}{x} = -f(x)$$

f est impaire sym / 0

$$f(3) = 1$$

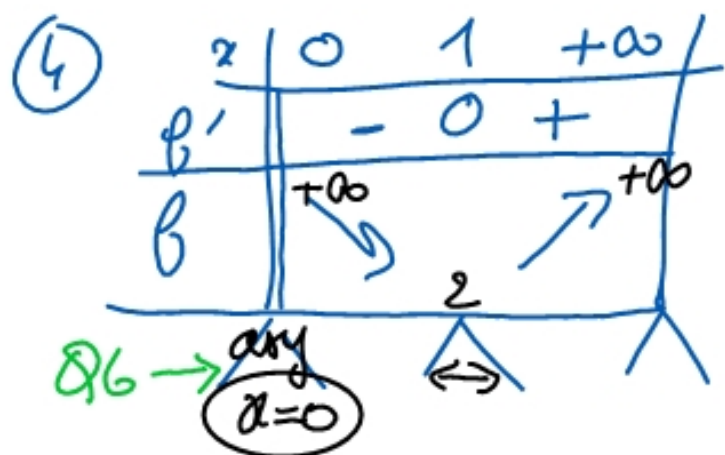
$$f(-3) = -1$$



(3) f est dérivable pour $x \neq 0$ comme quotient de (den $\neq 0$) fonction . .

$$f'(x) = \frac{2x^2 - (x^2+1)}{x^2} = \frac{x^2-1}{x^2}$$

$$f'(x) = 0 \Leftrightarrow x^2 - 1 = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$$



$$f(1) = 2$$

$$\lim_{0^+} f(x) = \lim_{0^+} \frac{x^2+1}{0} = +\infty \quad \left(\frac{1}{0} = \infty\right)$$

$$\lim_{+\infty} f(x) = \lim_{+\infty} \frac{x^2}{x} = \lim_{+\infty} x = +\infty$$

⑤ $f'(1) = 0$ $f(1) = 2$ $\begin{matrix} \text{pente} \\ \uparrow \\ \text{pente} \end{matrix}$ $\begin{matrix} \text{pente} \\ \downarrow \\ \text{tangente} \end{matrix}$ $y = ax + b$

$y = 2$ \leftarrow $f'(1) = 0$

tpa: $y = f(a) + f'(a)(x-a)$

en 2: $y = f(2) + f'(2)(x-2)$

$f(2) = \frac{5}{2}$ $f'(2) = \frac{3}{4}$

$y = \frac{5}{2} + \frac{3}{4}(x-2)$

$y = \frac{3}{4}x - \frac{1}{2}$ \leftarrow $f'(2) = \frac{3}{4}$

⑦ en $+\infty$ $f(x) \rightarrow +\infty$

$a = \lim_{+\infty} \frac{f(x)}{x} = \lim_{+\infty} \frac{x^2+1}{x^2} = \lim_{+\infty} \frac{x^2}{x^2} = 1$

$b = \lim_{+\infty} (y - ax) = \lim_{+\infty} \frac{x^2+1}{x} - x = \lim_{+\infty} \frac{x^2+1-x^2}{x} = \lim_{+\infty} \frac{1}{x} = 0$

$\boxed{\text{asy } y = x}$

