

① racine $n^{\text{ième}}$ d'un complexe
Les racines $n^{\text{ième}}$ de $Re^{i\theta}$ sont les $z_k = R^{\frac{1}{n}} e^{i\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)}$ où $k=0, 1, \dots, n-1$

② $\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \sin(\beta)\cos(\alpha)$

③ dérivée de $\arctan x$
 $\arctan' x = \frac{1}{1+x^2}$

④ DL en 0 à l'ordre n de $\frac{1}{1+x}$
 $\frac{1}{1+x} = 1 - x + x^2 - \dots + (-1)^n x^n + o(x^n) = \sum_{k=0}^n (-1)^k x^k + o(x^n)$

⑤ $\cos(2x) = \overset{?}{2}\overset{?}{\cos^2} x - 1 = 1 - 2\sin^2 x$

$$\textcircled{1} \int e^x \cos(e^x) dx = I \quad \text{chgt } X = e^x$$

$$\frac{dX}{dx} = e^x \Rightarrow dx = \frac{dX}{e^x} = \frac{dX}{X}$$

$$I = \int \cancel{X} \cos X \frac{dX}{\cancel{X}} = \int \cos X dX = \sin X + \text{cte} = \underline{\sin(e^x) + \text{cte}}$$

$$\textcircled{2} I = \int \frac{\sin(\ln x)}{x} dx \quad \text{chgt } X = \ln x \quad \frac{dX}{dx} = \frac{1}{x} \Rightarrow \underline{dx = x dX}$$

$$I = \int \sin(\cancel{X}) \cancel{x} dX = -\cos X + \text{cte} = \underline{-\cos(\ln x) + \text{cte}}$$

$$\textcircled{6} I = \int \frac{x}{(x^2+1)^3} dx \quad \text{chgt } X = x^2+1 \quad \frac{dX}{dx} = 2x \Rightarrow dx = \frac{dX}{2x}$$

$$I = \int \frac{\cancel{x}}{X^3} \frac{dX}{2\cancel{x}} = \frac{1}{2} \int X^{-3} dX = \frac{1}{2} \frac{X^{-2}}{-2} + \text{cte} = \underline{-\frac{1}{4(x^2+1)^2} + \text{cte}}$$

$$\textcircled{3} \quad I = \int \cos x \sin^3 x \, dx \quad \text{chgt } X = \sin x \quad \frac{dX}{dx} = \cos x$$

$$\Rightarrow dx = \frac{dX}{\cos x} \quad I = \int \cancel{\cos x} X^3 \frac{dX}{\cancel{\cos x}} = \frac{X^4}{4} + cte = \underline{\underline{\frac{\sin^4 x}{4} + cte}}$$

$$\textcircled{4} \quad \int 37 \, dx = 37x + cte \quad \int 37 \, dt = 37t + cte$$

$$\int \cos x \sin^3 x \, dx = \cos x \sin^3 x \, x + cte$$

$$\textcircled{5} \quad I = \int t e^{t^2} \, dt \quad \text{chgt } X = t^2 \quad \frac{dX}{dt} = 2t \Rightarrow dt = \frac{dX}{2t}$$

$$I = \int \cancel{t} \frac{e^X}{2\cancel{t}} \, dX = \frac{1}{2} e^X + cte = \frac{1}{2} e^{t^2} + cte$$

$$\textcircled{7} \quad \int \frac{\cos t}{\sin^5 t} \, dt = I \quad \text{chgt } X = \sin t \quad \frac{dX}{dt} = \cos t \Rightarrow dt = \frac{dX}{\cos t}$$

$$I = \int \frac{\cancel{\cos t}}{X^5} \frac{dX}{\cancel{\cos t}} = \int X^{-5} \, dX = \frac{X^{-4}}{-4} + cte = \underline{\underline{-\frac{1}{4 \sin^4 t} + cte}}$$

$$\textcircled{8} \int \frac{\cot t}{\sin^2 t + 1} dt = I \quad \text{dykt } X(t) = \sin t \quad \frac{dX}{dt} = \cos t \Rightarrow dt = \frac{dX}{\cos t}$$

$$I = \int \frac{\cancel{\cot}}{X^2 + 1} \frac{dX}{\cancel{\cos t}} = \arctan(X) + C_0 = \arctan(\sin t) + C_0$$

$$\textcircled{9} \int e^{2t+1} dt = I \quad \text{dykt } X = 2t+1 \quad \frac{dX}{dt} = 2$$

$$\Rightarrow dt = \frac{dX}{2} \quad I = \int e^X \frac{dX}{2} = \frac{1}{2} e^X = \frac{e^{2t+1}}{2}$$

$$\textcircled{10} I = \int \frac{e^x}{1+e^{2x}} dx \quad \text{dykt } X = e^x \quad \frac{dX}{dx} = e^x \Rightarrow dx = \frac{dX}{e^x}$$

$$I = \int \frac{\cancel{e^x}}{1+X^2} \frac{dX}{\cancel{e^x}} \quad \text{cau } e^{2x} = (e^x)^2 = X^2$$

$$= \arctan(X) + C_0 = \arctan(e^x) + C_0$$

Distance d'un pt à un plan

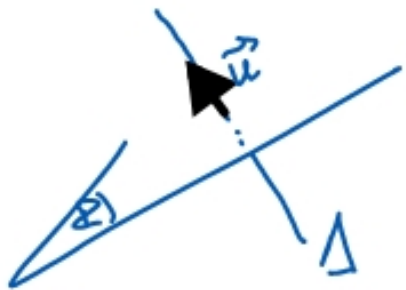
exemple $\mathcal{P}: 2x + y - 3z = 2$

plan vectoriel associé : $\vec{\mathcal{P}}: 2x + y - 3z = 0$

un vecteur normal à \mathcal{P} : $(2, 1, -1) = \vec{u}$

3 vecteurs de $\vec{\mathcal{P}}$: $\vec{u}(1, 0, 2)$ et $\vec{v}(0, 0, 0)$ et $\vec{w}(0, 1, 1)$

$A(2, 1, 4) \in \mathcal{P}$? $2x_A + y_A - 3z_A = 4 + 1 - 4 = 1 \neq 2$, donc $A \notin \mathcal{P}$



eq param de Δ $\left\{ \begin{array}{l} x = 2t + 2 \\ y = 1t + 1 \text{ où } t \in \mathbb{R} \\ z = -t + 4 \end{array} \right.$

$$\text{Intersection de } \mathcal{L} \text{ et } \Delta: B \in \mathcal{L} \left. \begin{array}{l} 2x_B + y_B - z_B = 2 \\ B \in \Delta \end{array} \right\} \begin{array}{l} x_B = 2t_B + 2 \\ y_B = t_B + 1 \\ z_B = -t_B + 4 \end{array} \quad \nearrow$$

$$2(2t_B + 2) + (t_B + 1) - (-t_B + 4) = 2 \Rightarrow 6t_B + 1 = 2 \Rightarrow 6t_B = 1$$

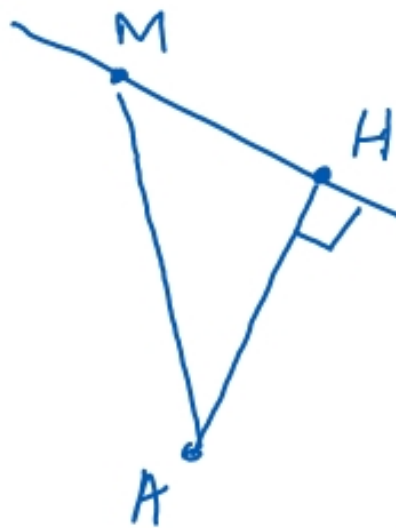
$$t_B = \frac{1}{6} \quad x_B = \frac{2}{6} + 2 = \frac{14}{6} \quad y_B = \frac{1}{6} + 1 = \frac{7}{6}; \quad z_B = -\frac{1}{6} + 4 = \frac{23}{6}$$

$$B = \left(\frac{14}{6}, \frac{7}{6}, \frac{23}{6} \right)$$

$$\vec{AB} = B - A = \left(\frac{14}{6}, \frac{7}{6}, \frac{23}{6} \right) - (2, 1, 4) = \left(\frac{2}{6}, \frac{1}{6}, -\frac{1}{6} \right)$$

$$\|\vec{AB}\| = \frac{1}{6} \|(2, 1, -1)\| = \frac{1}{6} \sqrt{2^2 + 1^2 + (-1)^2} = \frac{\sqrt{6}}{6}$$

$$\frac{|2x_A + y_A - z_A - 2|}{\sqrt{2^2 + 1^2 + (-1)^2}} = \frac{|4 + 1 - 4 - 2|}{\sqrt{6}} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$



$$(D) \begin{cases} x=1+2t \\ y=3-t \\ z=2+3t \end{cases} \quad A = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

$$\vec{AM} = \begin{pmatrix} x-2 \\ y-3 \\ z+1 \end{pmatrix} = \begin{pmatrix} 2t-1 \\ -t \\ 3t+3 \end{pmatrix}$$

$$AM = \|\vec{AM}\| = \sqrt{(2t-1)^2 + (-t)^2 + (3t+3)^2}$$

$$= \sqrt{4t^2 - 4t + 1 + t^2 + 9t^2 + 18t + 9} = \sqrt{14t^2 + 14t + 10} = f(t)$$

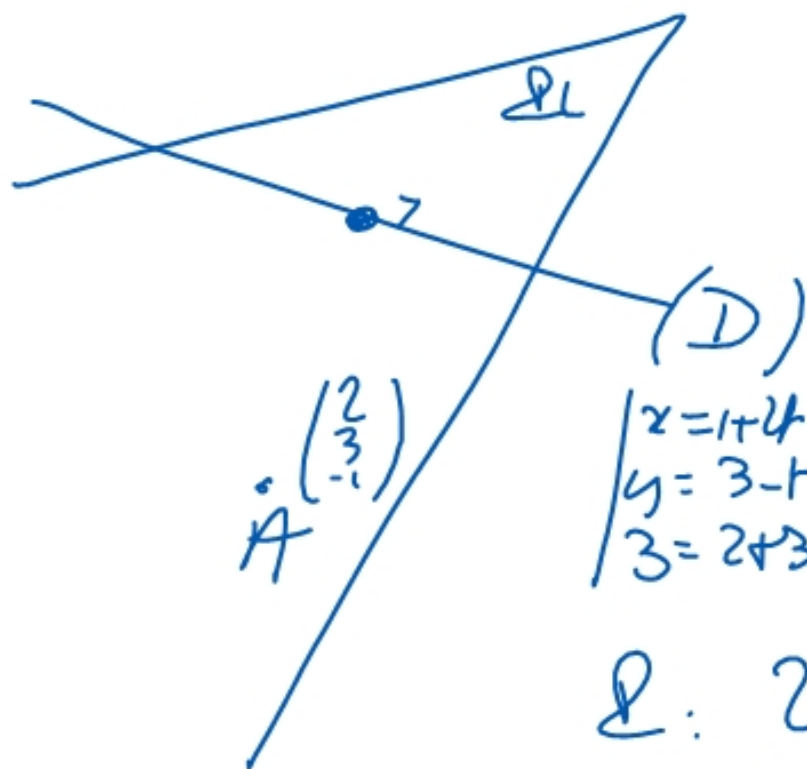
$$f'(t) = \frac{1}{2\sqrt{\quad}} \cdot (28t + 14) \quad f'(t) = 0 \Leftrightarrow 28t + 14 = 0$$

$$\Rightarrow t_H = -\frac{14}{28} = -\frac{1}{2}$$

$$x_H = 1 + 2t_H = 1 - 1 = 0 \quad y_H = 3 + \frac{1}{2} = \frac{7}{2} \quad z_H = 2 - \frac{3}{2} = \frac{1}{2}$$

$$H = \left(0; \frac{7}{2}; \frac{1}{2}\right) \quad \vec{AH} = \left(-2; -\frac{1}{2}; \frac{3}{2}\right) \quad AH = \sqrt{4 + \frac{1}{4} + \frac{9}{4}}$$

$$AH = \frac{\sqrt{26}}{2} = \sqrt{\frac{13}{2}}$$



P passe par A et est \perp à (D)

Une vecteur directeur de (D) ,

$$\vec{u} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \perp P$$

$$P: 2x - y + 3z = c$$

$$4 - 3 - 3 = c = -2$$

P passe par A

$$P: 2x - y + 3z - 2 = 0$$

Exemple 3 :

$$\begin{cases} x(t) = \tan t + \sin t \\ y(t) = \frac{1}{\cos(t)} \end{cases}$$

$$D_f = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi \text{ où } k \in \mathbb{Z} \right\}$$

car $\cos t = 0$
 lorsque $t = \frac{\pi}{2} + k\pi$

période :

$$\begin{aligned} x(t + 2\pi) &= \tan(t + 2\pi) + \sin(t + 2\pi) \\ &= \tan(t) + \sin t = x(t) \\ y(t + 2\pi) &= y(t) \end{aligned} \quad \left. \vphantom{\begin{aligned} x(t + 2\pi) \\ y(t + 2\pi) \end{aligned}} \right) \text{ donc } M(t + 2\pi) = M(t)$$

Etude sur $[-\pi, \pi] \setminus \{-\frac{\pi}{2}, \frac{\pi}{2}\}$. Mais sur toute la courbe!

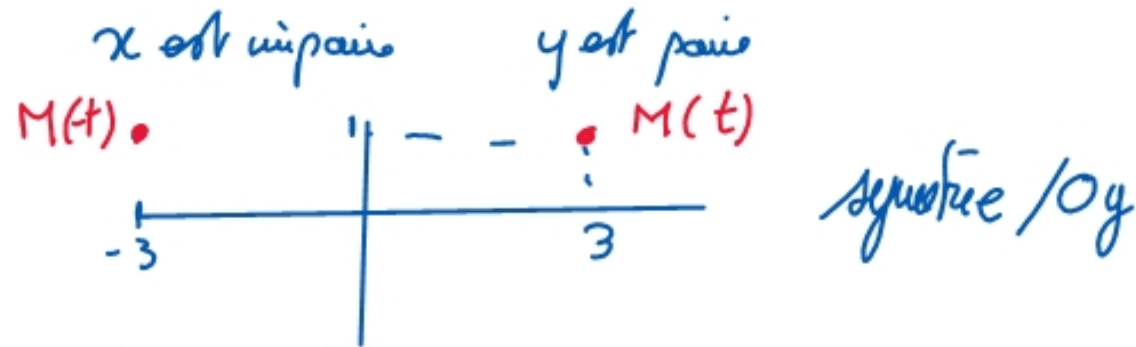
parité :

$$x(-t) = \tan(-t) + \sin(-t) = -\tan(t) - \sin(t) = -(\tan t + \sin t) = -x(t)$$

x est impaire

$$y(-t) = \frac{1}{\cos(-t)} = \frac{1}{\cos(t)} = y(t) \quad y \text{ est paire}$$

$$\begin{cases} x = \tan t + \frac{1}{\cos t} \\ y = \frac{1}{\cos t} \end{cases}$$



Etude sur $[0, \pi] \setminus \{\frac{\pi}{2}\}$ + symétrique / Oy.

$$x'(t) = \frac{1}{\cos^2 t} + \cos t = \frac{1 + \cos^3 t}{\cos^2 t} \quad \text{de signe de } 1 + \cos^3 t \geq 0, \text{ nul en } \pi$$

$$y'(t) = -\frac{1}{\cos^2 t} \times (-\sin t) \quad \text{car } \left(\frac{1}{u}\right)' = -\frac{1}{u^2} \cdot u'$$

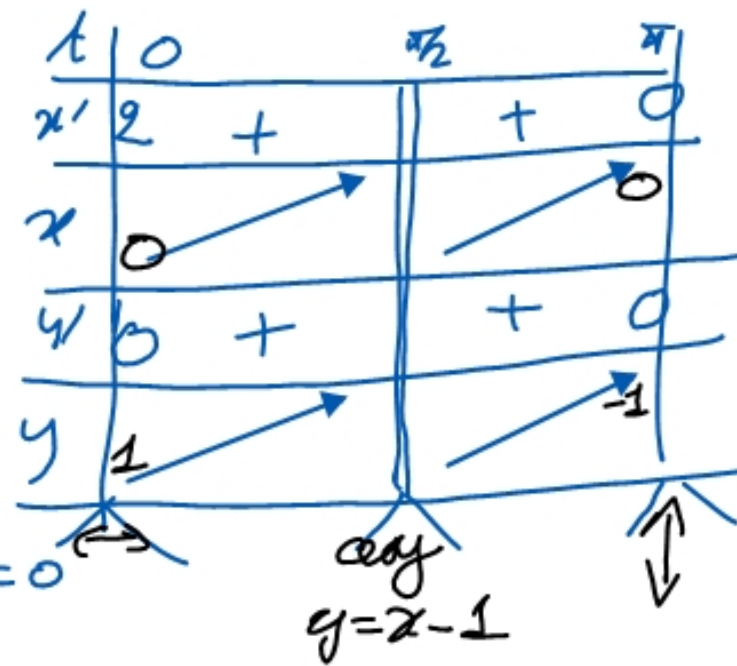
$$y'(t) = \frac{\sin t}{\cos^2 t} \quad y' \text{ est du signe de } \sin t \geq 0, \text{ nul en } 0 \text{ et en } \pi$$

$$\begin{cases} x = \cos t + \sin t & x' = \frac{1 + \cos^3 t}{\cos^2 t} \\ y = \frac{1}{\cos t} & y' = \frac{\sin t}{\cos^2 t} \end{cases}$$

en 0: $x(0) = 0 + 0 = 0$
 $y(0) = \frac{1}{1} = 1$

$M(0) = (0, 1)$ $x'(0) = 2$; $y'(0) = 0$

$\vec{T}(0) = (2, 0)$ horizontal



en π : $x(\pi) = 0 + 0 = 0$ $y(\pi) = \frac{1}{-1} = -1$

$x'(\pi) = y'(\pi) = 0$ point singulier

La pente vaut $p = \lim_{t \rightarrow \pi} \frac{y'}{x'} = \lim_{t \rightarrow \pi} \frac{\frac{\sin t}{\cos^2 t}}{\frac{1 + \cos^3 t}{\cos^2 t}} = \lim_{t \rightarrow \pi} \frac{\sin t}{1 + \cos^3 t} = \frac{0}{0}$ ZUT

Chgt $T = t - \pi \Rightarrow t = T + \pi$

$\lim_{T \rightarrow 0} \frac{\sin(T + \pi)}{1 + \cos^3(T + \pi)} = \lim_{T \rightarrow 0} \frac{-\sin(T)}{1 - \cos^3(T)} = \lim_{T \rightarrow 0} \frac{\sin(T)}{\cos^3(T) - 1}$

$$\frac{\sin T}{\cos^2(T)-1} = \frac{T + o(T)}{(1 + o(T))^3 - 1} = \frac{T + o(T)}{o(T)} = \frac{T(1 + o(1))}{T o(1)} \rightarrow \frac{1}{0} = \infty$$

peut être ∞ . tangente verticale

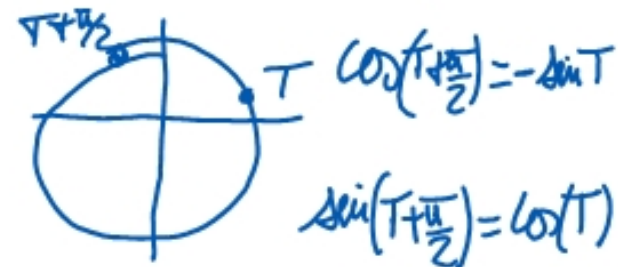
en $\frac{\pi}{2}$ $x(t) \rightarrow \infty$ et $y(t) \rightarrow \infty$ asymptote oblique?

$$a = \lim_{T \rightarrow \frac{\pi}{2}} \frac{y}{x} = \lim_{T \rightarrow \frac{\pi}{2}} \frac{\frac{1}{\cos T}}{\frac{\sin T}{\cos T} + \sin T} \times \frac{\cos T}{\cos T} = \lim_{T \rightarrow \frac{\pi}{2}} \frac{1}{\sin T + \sin T \cos T} = \frac{1}{1+0} = 1$$

$a=1$

$$b = \lim_{T \rightarrow \frac{\pi}{2}} y - ax = \lim_{T \rightarrow \frac{\pi}{2}} \frac{1}{\cos T} - \left(\frac{\sin T}{\cos T} + \sin T \right) = \lim_{T \rightarrow \frac{\pi}{2}} \frac{1 - \sin T - \sin T \cos T}{\cos T} = \frac{0}{0} \text{ ZUT}$$

$$\text{d'après } T = t - \frac{\pi}{2} \Rightarrow t = T + \frac{\pi}{2}$$



$$b = \lim_{T \rightarrow 0} \frac{1 - \sin(T + \frac{\pi}{2}) - \sin(T + \frac{\pi}{2}) \cos(T + \frac{\pi}{2})}{\cos(T + \frac{\pi}{2})}$$

$$b = \lim_{T \rightarrow \infty} \frac{1 - \cos(T) + \sin T \cos T}{-\sin T} = \lim_{T \rightarrow \infty} \frac{1 - 1 + T \cdot 1 + o(T)}{-T + o(T)} = -1$$

asymptote $y = x - 1$

