

$$\textcircled{1} \binom{11}{2} = \frac{11!}{2!9!} = \frac{\cancel{2} \cdot \cancel{3} \dots \cancel{9} \cdot 10 \cdot 11}{2 \cdot \cancel{2} \cdot \cancel{3} \dots \cancel{9}} = \frac{10 \cdot 11}{2} = 5 \cdot 11 = 55$$

\textcircled{2} DL de $\ln(1+x)$, à l'ordre n

$$\ln(1+x) = x - \frac{x^2}{2} + \dots + (-1)^{\overbrace{n+1}^{\text{ou } n-1}} \frac{x^n}{n} + o(x^n) \quad \text{car } (-1)^{n+1} = (-1)^{n-1} \cdot (-1)^2 = (-1)^{n-1}$$

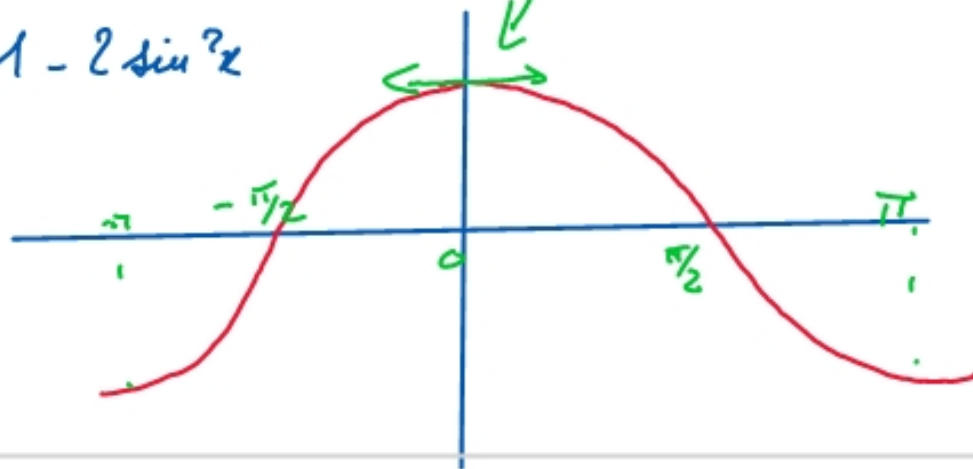
\textcircled{3} dérivée de arctan x

$$\text{arctan}' x = \frac{1}{1+x^2}$$

\textcircled{4} $\cos(2x) = ?$ 2 façons \neq

$$\cos(2x) = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

\textcircled{5} dessin de $\cos(x)$



$$\textcircled{1} \sqrt{x}' = x^{\frac{1}{2}'} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \quad (x^n)' = n x^{n-1}$$

$$\sqrt{2x+3}' = \frac{1}{2\sqrt{2x+3}} \cdot 2 \quad \text{car } (u^n)' = n u^{n-1} \cdot u'$$

$$(x^5)' = 5x^4 \quad \sqrt{x^5}' = (x^5)^{\frac{1}{2}'} = x^{\frac{5}{2}'} = \frac{5}{2} x^{\frac{3}{2}}$$

$$\left(\frac{x^3}{\sqrt{x}}\right)' = x^{3-\frac{1}{2}'} = x^{\frac{5}{2}'} = \frac{5}{2} x^{\frac{3}{2}} \quad \ln(\sin x)' = \frac{1}{\sin x} \cdot \cos x$$

$$\ln(\sin 2x)' = \frac{1}{\sin(2x)} \cdot \cos(2x) \cdot 2$$

$$\text{car } \sin(2x)' = \cos(2x) \cdot 2$$

$$\ln(\sin(x^2))' = \frac{1}{\sin(x^2)} \cdot \cos(x^2) \cdot 2x$$

$$\ln(\sin^2 x) = \frac{1}{\sin^2 x} \cdot 2 \sin x \cdot \cos x \quad \text{cas } e^{u^2} = 2e \cdot u'$$



$$\sin(\ln x)' = \cos(\ln x) \cdot \frac{1}{x}$$

$$\sin(\sqrt{x}) = \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} \quad \sin(\ln(x^2+1))' = \cos(\ln(x^2+1)) \cdot \frac{1}{x^2+1} \cdot 2x$$

$$\sin(\sqrt{\ln x})' = \cos(\sqrt{\ln x}) \cdot \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}$$

$$\sin^5 x' = 5 \sin^4 x \cdot \cos x$$

$$\text{cas } e^5 = 5u^4 \cdot u'$$

$$\sin^5(x^2) = 5 \sin^4(x^2) \cdot \underbrace{\cos(x^2) \cdot 2x}_{\text{derivada de } \sin(x^2)}$$

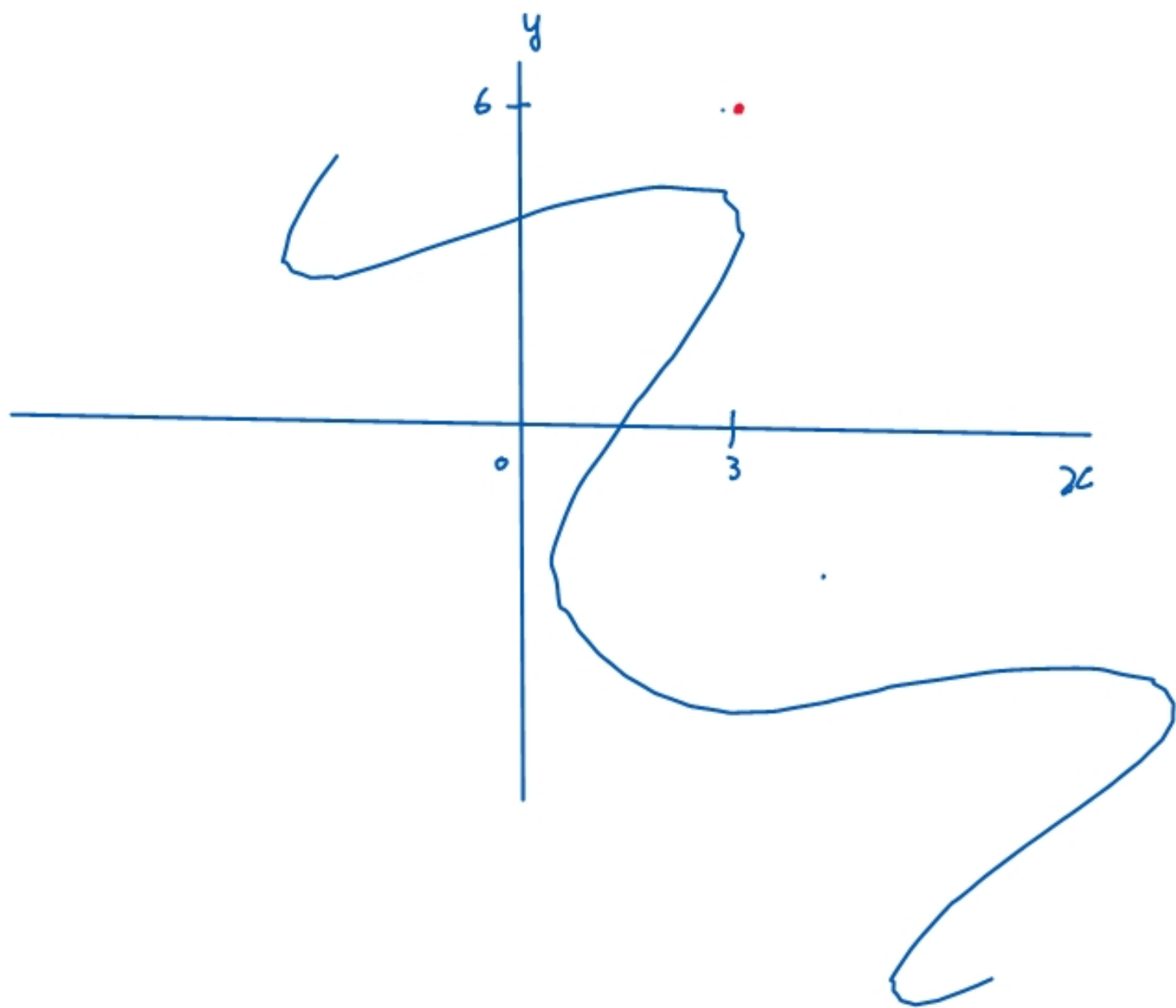
$$\sin(x^5)' = \cos(x^5) \cdot 5x^4$$

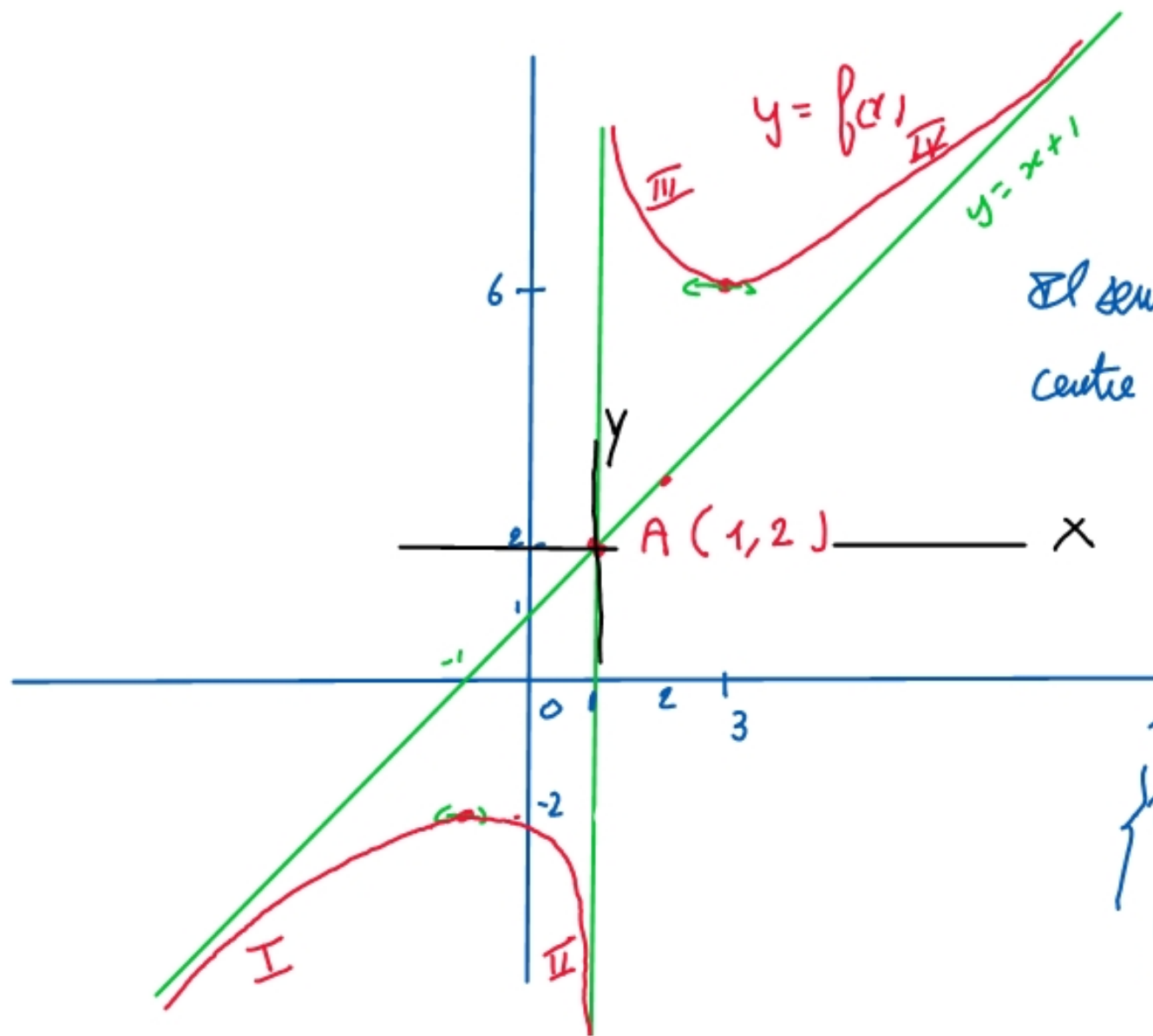
$$\sin^5(\ln(x^2+1))' = 5 \sin^4(\ln(x^2+1)) \cdot \cos(\ln(x^2+1)) \cdot \frac{1}{x^2+1} \cdot 2x$$

$$\tan(2x)' = \frac{1}{\cos^2(2x)} \cdot 2$$

$$\tan(\sqrt{x})' = \frac{1}{\cos^2(\sqrt{x})} \cdot \frac{1}{2\sqrt{x}}$$

$$\tan(\ln(x^2+1))' = \frac{1}{\cos^2(\ln(x^2+1))} \cdot \frac{1}{x^2+1} \cdot 2x$$





Il semble que A est
centre de symétrie

On fait un changement
de repère : on cherche
la fonction dans le

repère (A, X, Y)

$$x = X + 1$$

$$y = Y + 2$$

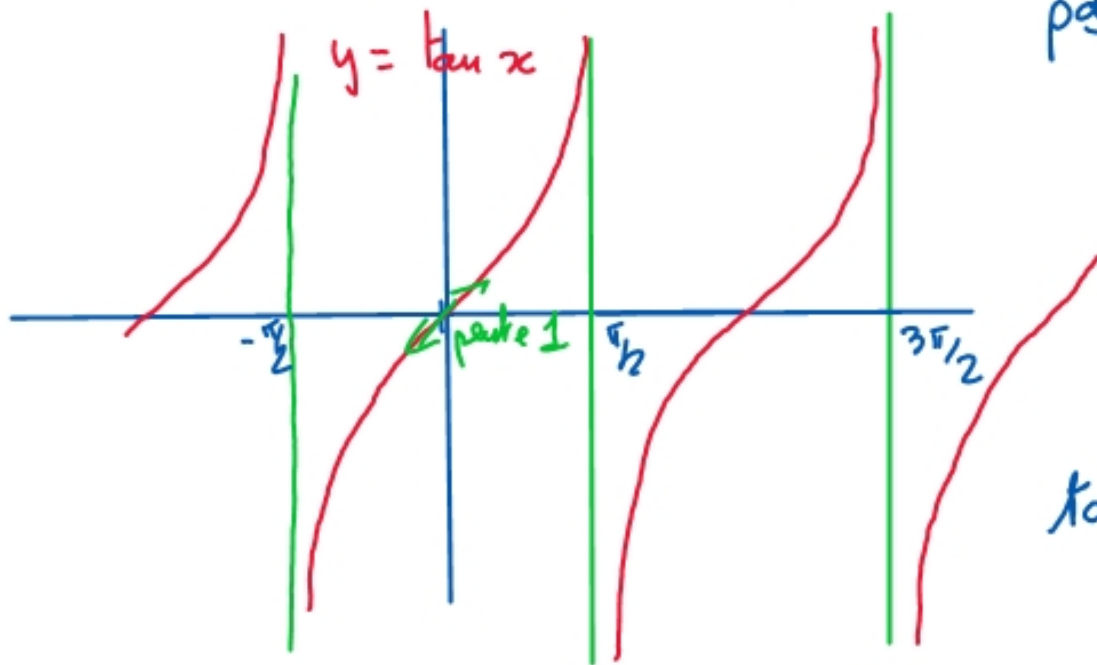
$$y = f(x) \Leftrightarrow Y + 2 = \frac{x^2 + 3}{x - 1} = \frac{(X + 1)^2 + 3}{X + 1 - 1} = \frac{X^2 + 2X + 4}{X} \Rightarrow Y = \frac{X^2 + 2X + 4}{X} - 2$$

$$y = \frac{x^2 + 2x + 4 - 2x}{x} = \frac{x^2 + 4}{x} = g(x),$$

$$g(-x) = \frac{(-x)^2 + 4}{-x} = -\frac{x^2 + 4}{x} = -g(x). \quad g \text{ est } \underline{\text{impaire}}$$

Donc, il y a une symétrie / A

VII les fonctions classiques

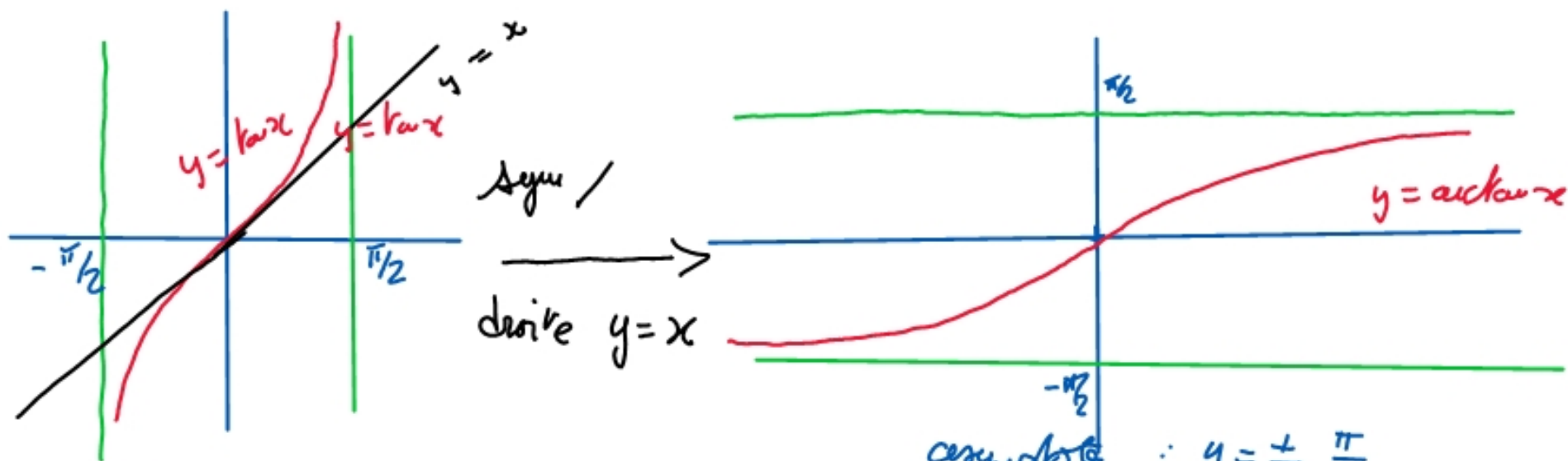


$$\underline{\text{paire}}: \tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan x$$

\tan est impaire, donc symétrie / O

période

$$\tan(x + \pi) = \frac{\sin(x + \pi)}{\cos(x + \pi)} = \frac{-\sin x}{-\cos x} = \tan x \quad \text{période } \pi$$



ch et sh : $\text{ch}(x) = \frac{e^x + e^{-x}}{2}$

asymptotes : $y = \pm \frac{\pi}{2}$
 $\text{sh}(x) = \frac{e^x - e^{-x}}{2}$

parité : $\text{ch}(-x) = \frac{e^{-x} + e^x}{2} = \text{ch}(x)$. ch est paire .

Ekudo sur \mathbb{R}^+ + symétrie / Oy

$\text{sh}(-x) = \frac{e^{-x} - e^x}{2} = -\frac{(e^x - e^{-x})}{2} = -\text{sh}(x)$. sh est impair

$\text{ch}'(x) = \frac{e^x - e^{-x}}{2} = \text{sh}(x)$

$\text{sh}'(x) = \frac{e^x + e^{-x}}{2} = \text{ch}(x)$
 > 0

x	0	$+\infty$
f'	1	$+$
f	0	\nearrow

$\frac{e^x - e^{-x}}{2x} = \frac{e^x(1 - e^{-2x})}{2x} \rightarrow 0$

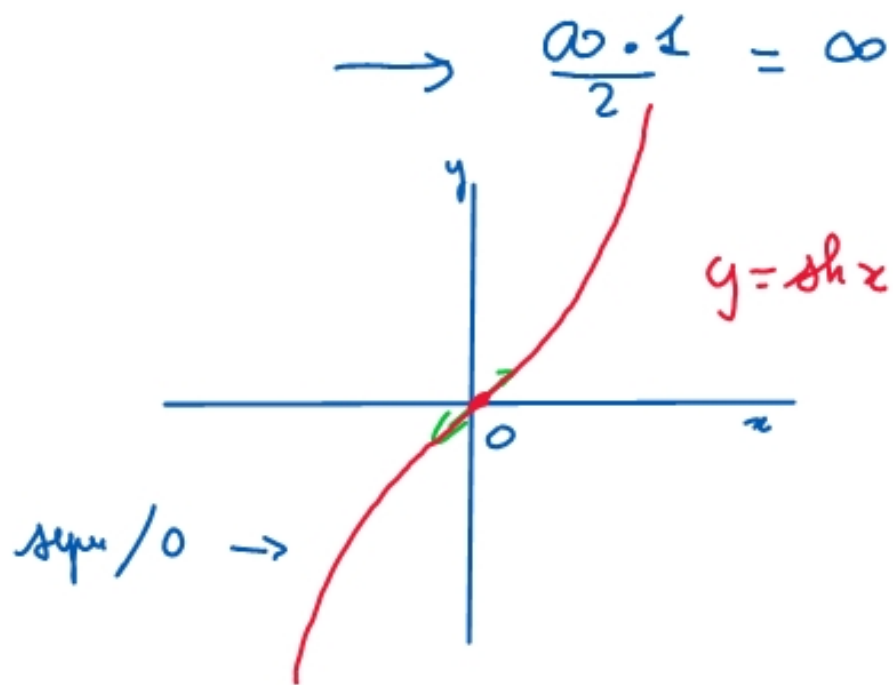
$$\text{sh}(0) = \frac{e^0 - e^0}{2} = 0$$

$$\text{sh}'(0) = \text{ch}(0) = \frac{e^0 + e^0}{2} = 1$$

$\frac{e^{+\infty}}{+\infty}$: asymptote ?

$$a = \lim_{x \rightarrow +\infty} \frac{y}{x} = \lim_{x \rightarrow +\infty} \frac{\text{sh}x}{x} = \lim_{x \rightarrow +\infty} \frac{e^x - e^{-x}}{2x} = \frac{\infty}{\infty}$$

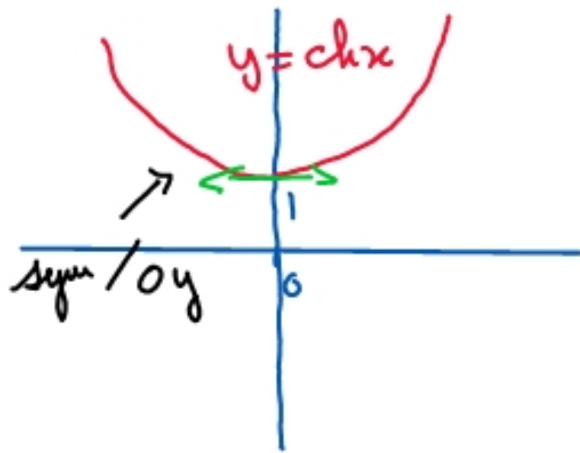
$$\lim_{x \rightarrow +\infty} \frac{e^x}{x} = \lim_{x \rightarrow +\infty} e^x = +\infty$$



Pas de droite asymptote

x	0	$+\infty$
ch'	0	$+$
ch	\nearrow	\nearrow

m calcul que pour sh
 pas de droite asymptote



$$\begin{aligned} \operatorname{ch}(x) - \operatorname{sh}(x) &= \frac{e^x + e^{-x}}{2} - \frac{e^x - e^{-x}}{2} \\ &= \frac{e^x + e^{-x} - e^x + e^{-x}}{2} = \frac{2e^{-x}}{2} = e^{-x} \end{aligned}$$

$\operatorname{ch}x - \operatorname{sh}x > 0$ car $\exp(x) > 0 \forall x$
de plus, $\operatorname{ch}x - \operatorname{sh}x \xrightarrow{+\infty} 0$

donc ch et sh sont asymptote en $+\infty$



VIII Concavité

$f'' > 0$
concave

convexe
 $f'' < 0$

Le point d'inflexion est $f''(x_0) = 0$ et
 f'' change de signe

exo 6: $f(x) = \sin^3(2x)$ période π $T = \frac{2\pi}{\omega}$

Etude sur $[-\frac{\pi}{2}, \frac{\pi}{2}]$ + translations de π

parité: $f(-x) = \sin^3(2x(-x)) = \sin^3(-2x) = (-\sin(2x))^3 = -\sin^3(2x)$

$f(-x) = -f(x)$ fonction impaire Etude sur $[0, \frac{\pi}{2}]$ + sym/O

$f'(x) = 3\sin^2(2x) \cdot \cos(2x) \cdot 2$

$f'(x) = 0 \Leftrightarrow \begin{cases} \sin(2x) = 0 & 2x = k\pi & x = k\frac{\pi}{2} \\ \text{ou} & \\ \cos(2x) = 0 & 2x = \frac{\pi}{2} + k\pi & x = \frac{\pi}{4} + k\frac{\pi}{2} \end{cases}$ pour $x = 0; \frac{\pi}{4}; \frac{\pi}{2}$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
f'	0	+	0
f	0	↗	↘
	↖	<u>I</u>	<u>II</u>

