

$$\underline{(7-2x)}(3x+4) + 3(2x+1)\underline{(2x-7)} = A$$

$$(7-2x)(3x+4) - 3(2x+1)(7-2x) = A \quad \text{Car } -(7-2x) = 2x-7$$

$$A = (7-2x)[3x+4 - 3(2x+1)]$$

$$= (7-2x)[3x+4 - 6x - 3] = (7-2x)(-3x+1)$$

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méthode 2:  $A = 21x + 28 - 6x^2 - 8x + 3( \quad )$

$$= 6x^2 - 23x + 7$$

$$\Delta = 23^2 - 4 \times 6 \times 7 = 361 = 19^2$$

$$x = \frac{23 \pm 19}{12} = \begin{cases} \frac{4}{12} = \frac{1}{3} \\ \frac{42}{12} = \frac{7}{2} \end{cases}$$

$$\begin{array}{r} x = \\ 23 \\ \hline 69 \\ \hline 10 \\ \hline 129 \end{array}$$

$$A = 6x^2 - 23x + 7 \quad 2 \text{ racines } \frac{1}{3} \text{ et } \frac{7}{2}$$

On peut factoriser par  $x - \frac{1}{3}$  et  $x - \frac{7}{2}$

$$A = \underset{2 \times 3}{6} \left(x - \frac{1}{3}\right) \left(x - \frac{7}{2}\right)$$

$$= (3x - 1)(2x - 7)$$

$$5(7x^2+x) - (3x+8)(14x+2) = B$$

$$5x(7x+1) - (3x+8)2(7x+1) = B$$

$$(7x+1) [5x - (3x+8) \cdot 2] = (7x+1)(-x-16)$$

$$\begin{array}{r} 13 \\ -13 \\ \hline 139 \\ \hline 169 \end{array}$$

$$C = x^2 - 49 + (3x-4)(-x-7) + 2(2x+14)$$

$$= (x+7)(x-7) - (3x-4)(x+7) + 4(x+7)$$

$$= (x+7) [x-7 - (3x-4) + 4] = (x+7) [x-7-3x+4+4]$$

$$= \underbrace{(x+7)}_{-7} \underbrace{(-2x+1)}_{\frac{1}{2} \text{ soit racine}} = -2x^2 - 13x + 7$$

$$\Delta = 169 + 56 \\ = 225 = 5^2$$

$$\begin{aligned}
 (2x-6)^2 + (x-3) &= (2(x-3))^2 + (x-3) = 4(x-3)^2 + (x-3) \\
 &= (x-3) [4(x-3) + 1] = (x-3)(4x-11)
 \end{aligned}$$

$$9(4x^2 - 4x + 1) + (6x-3)x - 2x + 1 = \alpha$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$\begin{matrix} \uparrow & & \uparrow \\ 2x & & 1 \end{matrix}$

$$9(2x-1)^2 + 3(2x-1)x - (2x-1) = \alpha$$

$$\alpha = (2x-1) [9(2x-1) + 3x - 1] = (2x-1)(8x-10)$$

$$\begin{aligned}
 P(x) = x^3 + x^2 - x - 1 &= x^2(x+1) - (x+1) = (x+1)(x^2 - 1) \\
 &= (x+1)(x+1)(x-1) \\
 &= (x+1)^2(x-1)
 \end{aligned}$$

2<sup>ème</sup> méthode : On voit que 1 est racine. En effet  $P(1) = 1 + 1 - 1 - 1 = 0$   
 donc, on peut factoriser par  $x-1$ .

$$\begin{array}{r|l}
 x^3 + x^2 - x - 1 & x - 1 \\
 \underline{-(x^3 - x^2)} & x^2 + 2x + 1 \\
 2x^2 - x - 1 & \\
 \underline{-(2x^2 - 2x)} & \\
 x - 1 & \\
 \underline{-(x - 1)} & \\
 0 & 
 \end{array}$$

$$x^3 + x^2 - x - 1 = (x-1)(x^2 + 2x + 1) = (x-1)(x+1)^2$$

## V La formule du binôme de Newton

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

ou encore

$$(a+b)^3 = b^3 + 3b^2a + 3ba^2 + a^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(-b^4 + 4b^3a + 6b^2a^2 + 4ba^3 + a^4)$$

$$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$



$$\begin{array}{r}
 (a+b)^2 \rightarrow \quad \quad \quad 1 \quad 1 \\
 \quad \quad \quad \quad \quad 1 \quad 2 \quad 1 \\
 (a+b)^3 \quad \quad \quad 1 \quad 3 \quad 3 \quad 1 \\
 (a+b)^4 \rightarrow \quad 1 \quad 4 \quad 6 \quad 4 \quad 1 \\
 \quad \quad \quad 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
 \quad \quad \quad 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1
 \end{array}$$

Le triangle  
de Pascal

Les nombres s'appellent  
les combinaisons

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

$\binom{n}{p}$  est la place p dans la ligne n

cette combinaison se lit "p dans n"

$$\begin{array}{cccc}
 & \binom{1}{0} & \binom{1}{1} & \\
 & \binom{2}{0} & \binom{2}{1} & \binom{2}{2} \\
 \binom{3}{0} & \binom{3}{1} & \binom{3}{2} & \binom{3}{3} \\
 & \vdots & & 
 \end{array}$$

le triangle de  
Pascal

def 1:  $\binom{n}{p}$  est le nombre de parties à  $p$  éléments dans un ensemble à  $n$  éléments

def 2: si  $0 \leq p \leq n$

$$\binom{n}{p} = \frac{n!}{p! (n-p)!}$$

convention:

$$0! = 1$$

(convention: si  $p > n$   $\binom{n}{p} = 0$ )



$$\text{ex: } \binom{3}{1} = \frac{3!}{1!(3-1)!} = \frac{3!}{1!2!} = \frac{\cancel{2} \cdot 3}{1 \cdot 1 \cdot \cancel{2}} = 3$$

$$\binom{n}{1} = \frac{n!}{1!(n-1)!} = \frac{1 \cdot \cancel{2} \cdot \cancel{3} \dots (\cancel{n-1}) \cdot n}{1 \cdot \cancel{2} \cdot \cancel{3} \dots (\cancel{n-1})} = n$$

$$\binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{\cancel{n!}}{1 \cdot \cancel{n!}} = 1 \quad \binom{n}{n} = \frac{n!}{n!(n-n)!} = 1$$

$$\binom{n}{p} = \binom{n}{n-p} \quad \text{car } \binom{n}{n-p} = \frac{n!}{(n-p)! \underbrace{[n-(n-p)]!}_{p!}}$$

$$\boxed{\binom{n}{p} + \binom{n}{p+1} = \binom{n+1}{p+1}}$$

$$0 \leq p < n \quad \binom{n}{p} + \binom{n}{p+1} = \binom{n+1}{p+1}$$

$$\left[ \binom{n}{p} + \binom{n}{p+1} \right] = \frac{n!}{p!(n-p)!} + \frac{n!}{(p+1)!(n-p-1)!}$$

$\Delta \quad n - (p+1) = n - p - 1$

$$p!(p+1) = (p+1)! \quad \text{et} \quad (n-p-1)!(n-p) = (n-p)!$$

$$\binom{n}{p} + \binom{n}{p+1} = \frac{n!(p+1)}{\underbrace{p!(p+1)}_{(p+1)!} n-p)!} + \frac{n!(n-p)}{(p+1)! \underbrace{(n-p-1)!(n-p)}_{(n-p)!}}$$

$$= \frac{n!(p+1)}{(p+1)!(n-p)!} + \frac{n!(n-p)}{(p+1)!(n-p)!} = \frac{n! [p+1 + n-p]}{(p+1)!(n-p)!}$$

$$= \frac{n!(n+1)}{(p+1)!(n-p)!} = \frac{(n+1)!}{(p+1)!(n-p)!}$$

$$\text{or } \binom{n+1}{p+1} = \frac{(n+1)!}{(p+1)! \cdot [(n+1) - (p+1)]!} = \frac{(n+1)!}{(p+1)! \cdot (n-p)!}$$

$$\text{Hence } \binom{n}{p} + \binom{n}{p+1} = \binom{n+1}{p+1} \quad \square$$