

u

$$(21) \quad f(x) = \frac{2}{(3x-4)^3}$$

$$(u^m)' = m u^{m-1} \cdot u'$$

$$f(x) = 2(3x-4)^{-3}$$

$$(u^{-3})' = -3 u^{-4} \cdot u'$$

$$\Rightarrow f'(x) = -3(3x-4)^{-4} \cdot 3 \cdot 2 = \frac{-18}{(3x-4)^4}$$

$$(22) \quad f(x) = \ln(\cos(\pi x))$$

$$\ln(x)' = \frac{1}{x}$$

$$\ln(u(x))' = \frac{1}{u(x)} \cdot u'(x)$$

$$= \frac{1}{\cos(\pi x)} \cdot (-\sin(\pi x) \cdot \pi) = -\frac{\pi \sin(\pi x)}{\cos(\pi x)}$$

$$(23) \quad f(x) = \frac{1}{(x+1)(x+2)} = \frac{1}{x^2+3x+2}$$

$$(13) \quad f(x) = \frac{1}{(x-1)(x+2)} = \frac{1}{x^2+x-2}$$

$$\left(\frac{1}{x}\right)' = (x^{-1})' = -x^{-2} = -\frac{1}{x^2}$$

$$f'(x) = -\frac{1}{(x^2+x-2)^2} \cdot (2x+1) = \frac{-2x-1}{(x^2+x-2)^2}$$

$$\left(\frac{1}{u}\right)' = -\frac{1}{u^2} \cdot u'$$

$$(24) \quad f(x) = \frac{x}{\ln x}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{1 \cdot \ln x - x \cdot \frac{1}{x}}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$$

$$(25) \quad l = \lim_{x \rightarrow 1} \frac{(x-1)^2}{x^2-x} = \lim_{x \rightarrow 1} \frac{(x-1)^2}{x \cancel{(x-1)}} = \lim_{x \rightarrow 1} \frac{x-1}{x} = \frac{0}{1} = 0$$

ou bien def $X = x-1 \Rightarrow x = 1+X$

$$\frac{(x-1)^2}{x^2-x} = \frac{X^2}{(X+1)^2 - (X+1)} = \frac{X^2}{X^2+X} = \frac{X^2}{X(X+1)} \xrightarrow{x \rightarrow 0} \frac{0}{1} = 0$$

$$(26) \quad \lim_{-\infty} \frac{(x-1)^2}{x^2-x} = \lim_{-\infty} \frac{x^2-2x+1}{x^2-x} = \lim_{-\infty} \frac{x^2 (1 - \frac{2}{x} + \frac{1}{x^2})}{x^2 (1 - \frac{1}{x})}$$

$$= \frac{1}{1} = 1$$

$$(27) \quad \lim_{x \rightarrow 0} \frac{\cos(x - \frac{\pi}{2})}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{x + o(x)}{x} = \lim_{x \rightarrow 0} 1 + o(1) = 1$$

$$(28) \quad \lim_{x \rightarrow +\infty} (x^2-3)e^{-2x} = +\infty \times 0 \quad \text{F.I. ??}$$

$$\textcircled{29} \quad I = \int_0^2 \frac{2 dx}{(2x-3)^3}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\text{let } X = 2x - 3$$

$$\frac{dX}{dx} = 2 \Rightarrow dX = 2dx \Rightarrow dx = \frac{dX}{2}$$

$$I = \int_{x=-3}^{x=1} \frac{2}{X^3} \frac{dX}{2} = \int_{-3}^1 X^{-3} dX$$

$$= \left[\frac{X^{-2}}{-2} \right]_{-3}^1 = -\frac{1}{2} + \frac{(-3)^{-2}}{2} = -\frac{1}{2} + \frac{1}{2 \cdot 3^2} = \frac{-9+1}{18}$$

$$= -\frac{8}{18} = -\frac{4}{9}$$

$$(30) \int \frac{x+1}{3x^2+6x} dx = \frac{1}{6} \int \frac{6x+6}{3x^2+6x} dx = \frac{1}{6} \int \frac{u'}{u} = \frac{1}{6} \ln(3x^2+6x)$$

$$(31) \underline{I} = \int_{-1}^1 e^{|-x|} dx \quad \begin{array}{l} \text{qd } x > 0 \\ \text{qd } x \leq 0 \end{array} \quad \begin{array}{l} |-x| = x \\ |-x| = -x \end{array}$$

$$I = \int_{-1}^0 e^{-x} dx + \int_0^1 e^x dx$$

$$= [-e^{-x}]_{-1}^0 + [e^x]_0^1$$

$$= -e^0 + e^1 + e^1 - e^0 = 2e - 2$$

$$(32) \int \frac{x+1}{x^3+3x^2+6x} dx$$

on décompose en 21 simple

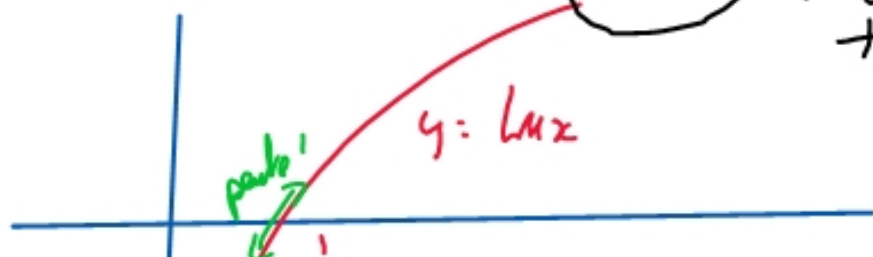
III les théorèmes de comparaison des croissances

Rappels :



$$\lim_{x \rightarrow -\infty} e^x = 0$$

$$\lim_{x \rightarrow +\infty} \ln(x) = +\infty$$



$$\lim_{x \rightarrow 0^+} \ln(x) = -\infty$$

$$\lim_{x \rightarrow +\infty} 2^x = +\infty$$

$$\lim_{x \rightarrow -\infty} 2^x = 0$$

$$\lim_{x \rightarrow +\infty} \frac{1}{2^x} = \frac{1}{\infty} = 0$$

$$\lim_{x \rightarrow +\infty} \left(\frac{1}{2}\right)^x = 0$$

$$\lim_{x \rightarrow +\infty} b^x = +\infty \text{ qd } b > 1$$

et

$$\lim_{x \rightarrow +\infty} b^x = 0 \text{ qd } b < 1$$

$$\boxed{\Pi_1} \quad \lim_{x \rightarrow +\infty} \frac{\ln(x)}{x^a} = 0 \quad (a > 0)$$

$\left(\frac{\infty}{\infty} \cdot \text{f.I.}\right)$

$$\boxed{\Pi_2} \quad \lim_{x \rightarrow 0^+} x^a \ln(x) = 0 \quad (a > 0)$$

$\left(0 \times (-\infty) \text{ f.I.}\right)$

On dit "les puissances l'emportent sur le Ln"

$$\boxed{\Pi_3} \quad \lim_{x \rightarrow +\infty} \frac{e^x}{x^a} = +\infty \quad (a > 0)$$

$\left(\frac{\infty}{\infty} \cdot \text{f.I.}\right)$

$$\boxed{\Pi_4} \quad \lim_{x \rightarrow -\infty} x^a e^x = 0 \quad (a > 0)$$

$\left(\infty \cdot 0 \text{ f.I.}\right)$

On dit "l'exponentielle l'emporte sur les puissances"

$$\ln(x) \ll x^n \ll e^x$$

$\sqrt{T_3}$

$$\lim_{+\infty} \frac{b^x}{x^a} = +\infty \quad (a > 0, b > 1)$$

← formule

$\sqrt{T_4}$

$$\lim_{-\infty} x^a b^x = 0 \quad (a > 0 \text{ et } b > 1)$$

ex: $\lim_{x \rightarrow +\infty} \frac{x^3 + 2e^x}{x^3 + \ln(x)}$

$$\left(\frac{\infty + \infty}{\infty + \infty} = \frac{\infty}{\infty} \text{ f. } \sqrt{\quad} \right)$$

$$\frac{x^3 + 2e^x}{x^3 + \ln(x)} = \frac{e^x (x^3 e^{-x} + 2)}{x^3 \left(1 + \frac{\ln x}{x^3} \right)}$$

$\rightarrow +\infty \frac{(0+2)}{(1+0)} = +\infty$

$\sqrt{T_3} \frac{e^x}{x^3} \xrightarrow{+\infty} +\infty$ (T des croissances comparées)
(TCC)

II Les équivalents

def: f est négligeable devant g au voisinage de x_0 lorsque $\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = 0$

exemple: $x^7 = o(e^x)$ notation $f = o(g)$ (rappel: $\frac{o(x^n)}{x^n} \rightarrow 0$)

def: f est équivalente à g au voisinage de x_0 lorsque $\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = 1$

notation: $f \sim_{x_0} g$

□ Si f est négligeable devant g , alors $f+g \sim_{x_0} g$

$$\left[f+g = g \left(\frac{f}{g} + 1 \right) \quad \frac{f+g}{g} = \frac{f}{g} + 1 \rightarrow 0 + 1 = 1 \right]$$

ex: $x^3 + 2x^2 - x + 5 \underset{+\infty}{\sim} x^3$

IT₂ Si f est équivalente à g en x_0 , alors $\lim_{x_0} f = \lim_{x_0} g$

IT₃ Si $\left. \begin{array}{l} f \sim_{x_0} F \\ g \sim_{x_0} G \end{array} \right\}$ alors $\left\{ \begin{array}{l} f \times g \sim_{x_0} F \times G \\ \text{et} \\ \frac{f}{g} \sim_{x_0} \frac{F}{G} \end{array} \right.$

⚠ $\left. \begin{array}{l} f+g \not\sim F+G \\ f-g \not\sim F-G \\ f(g) \not\sim F(G) \end{array} \right.$

ex 1: $\lim_{+\infty} \frac{x^3 + 2^x}{x^2 + 3^x} = \frac{\infty}{\infty}$ f. I

$$\left. \begin{array}{l} x^3 + 2^x \underset{+\infty}{\sim} 2^x \\ x^2 + 3^x \underset{+\infty}{\sim} 3^x \end{array} \right) \Rightarrow \frac{x^3 + 2^x}{x^2 + 3^x} \underset{+\infty}{\sim} \frac{2^x}{3^x} = \left(\frac{2}{3}\right)^x \xrightarrow{+\infty} 0$$

ex 2: $x^3 + 3x \underset{+\infty}{\sim} x^3$ et $x^3 + 3x \underset{0}{\sim} 3x$

car

$$\frac{x^3 + 3x}{x^3} = 1 + \frac{3}{x^2} \xrightarrow{+\infty} 1 + 0 = 1$$

ex 3: Équivalent en 0 de $\sin x - \tan x$

$$\sin x \underset{0}{\sim} x$$

$$\tan x \underset{0}{\sim} x$$

$$\sin x - \tan x \not\sim x - x = 0$$