

$$\frac{\ln(1+x)}{1+x} = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + O(x^4)}{1+x}$$

$$\begin{array}{r} x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + O(x^4) \\ -(x + x^2) \\ \hline \end{array}$$

$$-\frac{3}{2}x^2 + \frac{x^3}{3} - \frac{x^4}{4} + O(x^4)$$

$$-(-\frac{3}{2}x^2 - \frac{3}{2}x^3)$$

$$\frac{11}{6}x^3 - \frac{x^4}{4} + O(x^4)$$

$$-(\frac{11}{6}x^3 + \frac{11}{6}x^4)$$

$$-\frac{25}{12}x^4 + O(x^4)$$

$$-(-\frac{25}{12}x^4 + O(x^4))$$

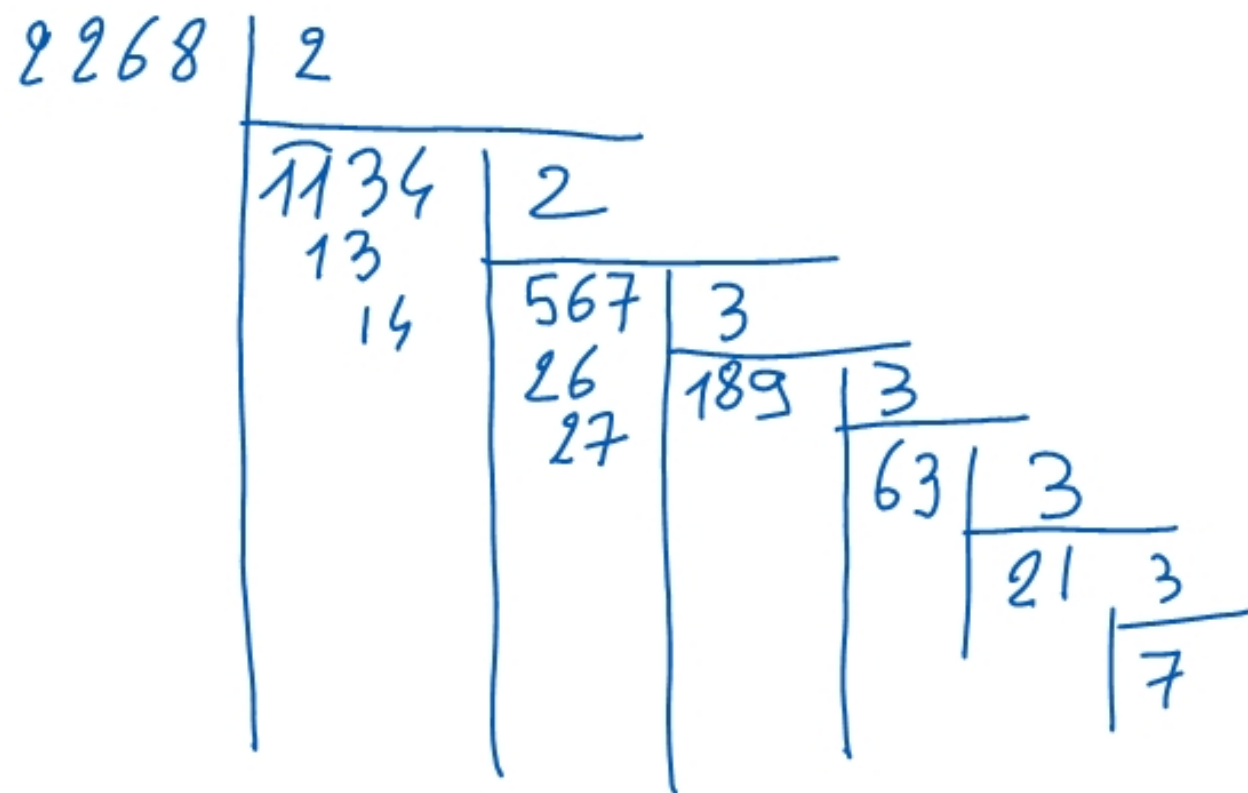
$$O(x^4)$$

$$1+x$$

$$x - \frac{3}{2}x^2 + \frac{11}{6}x^3 - \frac{25}{12}x^4 + O(x^4)$$

$$\frac{1}{3} + \frac{3}{2} = \frac{2}{6} + \frac{9}{6} = \frac{11}{6}$$

$$-\frac{1}{4} - \frac{11}{6} = \frac{-3}{12} - \frac{22}{12} = -\frac{25}{12}$$



$$2268 = 2^2 \times 3^4 \times 7$$

$$a = \frac{1}{3 + \frac{1}{\frac{1+1}{2+a}}} = \frac{1}{3 + \frac{1}{\frac{2+a+1}{2+a}} \leftarrow 3+a} = \frac{1}{3 + \frac{2+a}{3+a}}$$

$$= \frac{1}{\frac{3(3+a) + 2+a}{3+a}} = \frac{3+a}{9+3a+2+a} = \frac{3+a}{4a+11} = a$$

$$\Rightarrow 3+a = a(4a+11) = 4a^2 + 11a$$

$$4a^2 + 10a - 3 = 0$$

△
⋮
a

$$P(x) = x^3 - 21x + 20 \quad (31) \quad P(1) = 1 - 21 + 20 = 0, \text{ donc } \underline{\text{1 est racine}} \\ \underline{\text{de } P}$$

$$\begin{array}{r|l} x^3 - 21x + 20 & x - 1 \\ - (x^3 - x^2) & x^2 + x - 20 \\ \hline x^2 - 21x + 20 & \\ - (x^2 - x) & \\ \hline -20x + 20 & \\ - (-20x + 20) & \\ \hline 0 & \end{array}$$

$$P(x) = (x-1)(x^2 + x - 20)$$

$$\Delta = 81 = 9^2$$

$$? \text{ racines: } \frac{-1 \pm 9}{2} = \begin{matrix} -5 \\ 4 \end{matrix}$$

	$-\infty$	-5	$\frac{0}{20}$	1	2	4	$+\infty$
$P(x)$	$-$	0	$+$	0	$-$	0	$+$

$$P(x) = \text{Lm}(x^3 - 21x + 20)$$

\Rightarrow faut que $P(x) > 0$

$$\underline{D_p =]-5, 1[\cup]4, +\infty[}$$

$$\frac{-6}{x-3} < x+1 \Leftrightarrow \frac{-6}{x-3} - (x+1) < 0$$

$$\Leftrightarrow \frac{-6 - (x+1)(x-3)}{x-3} < 0$$

$$\Leftrightarrow \frac{-6 - (x^2 - 3x + x - 3)}{x-3} < 0 \Leftrightarrow \frac{-x^2 + 2x - 3}{x-3} < 0$$

$$-x^2 + 2x - 3 = 0$$

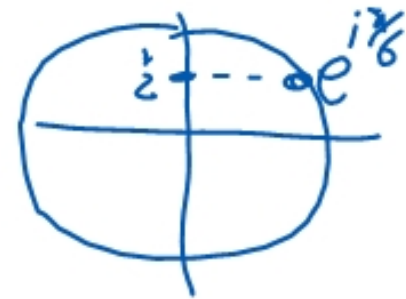
$$\Delta < 0.$$

	3	
$-x^2 + 2x - 3$	-	-
$x - 3$	-	+
$f(x)$	+	-

$$J =]3, +\infty[$$

$$z_1 = \frac{\sqrt{3}}{2} + \frac{i}{2} = e^{i\pi/6}$$

modulus 1, arg $\frac{\pi}{6}$



$$z_2 = \frac{\sqrt{3}}{2} - \frac{i}{2} = e^{-i\pi/6}$$

$$z_1 + z_2 = e^{i\pi/6} + e^{-i\pi/6} = 2\cos\left(\frac{\pi}{6}\right) = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$z_1^2 + z_2^2 = e^{i\pi/3} + e^{-i\pi/3} = 2\cos\left(\frac{\pi}{3}\right) = 2 \cdot \frac{1}{2} = 1$$

$$z_1^3 + z_2^3 = e^{i\pi/2} + e^{-i\pi/2} = i + (-i) = 0$$

$$z_1^{2023}$$

$$= e^{i \frac{2023\pi}{6}}$$

$$= e^{i(337\pi + \frac{\pi}{6})} = e^{i337\pi} \cdot e^{i\pi/6}$$

$$\begin{array}{r} 2023 \quad | \quad 6 \\ 22 \quad | \quad 337 \\ 43 \quad | \quad 1 \end{array}$$

$$2023 = 337 \times 6 + 1$$

$$\frac{2023}{6} = 337 + \frac{1}{6}$$

$$e^{-i\pi} \cdot e^{i\pi/6} = e^{i5\pi/6}$$

racines cubiques de i :

□ les racines n ièmes de $(Re^{i\theta})$ sont les $z_k = R^{\frac{1}{n}} e^{i(\frac{\theta}{n} + \frac{2k\pi}{n})}$
où $k=0, 1, \dots, n-1$

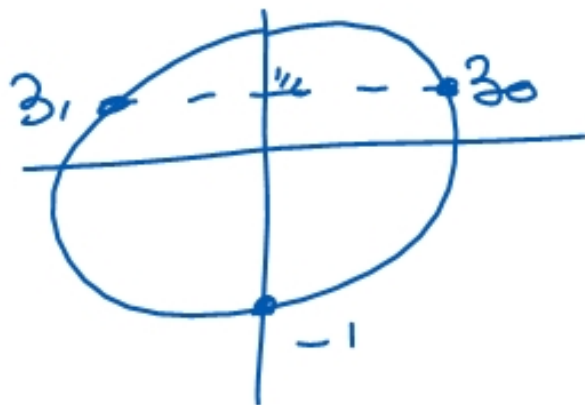
$i = e^{i\frac{\pi}{2}}$?

les racines 3 ièmes de $e^{i\frac{\pi}{2}}$ sont les $z_k = 1 e^{i(\frac{\pi/2}{3} + \frac{2k\pi}{3})}$
où $k=0, 1, 2$

$k=0: e^{i\frac{\pi}{6}} = z_0$

$k=1: z_1 = e^{i\frac{\pi}{6} + i\frac{2\pi}{3}} = e^{i\frac{5\pi}{6}}$

$z_2 = e^{i\frac{\pi}{6} + i\frac{4\pi}{3}} = e^{i\frac{9\pi}{6}} = e^{i\frac{3\pi}{2}} = -1$



$$-8 = 8e^{i\pi}$$

les racines 6^{es} de $8e^{i\pi}$ sont les $z_k = \left(8^{\frac{1}{6}}\right) e^{-i\left(\frac{\pi}{6} + \frac{4k\pi}{6}\right)}$ où $k=0, 1, \dots, 5$

$$8^{\frac{1}{6}} = (2^3)^{\frac{1}{6}} = 2^{\frac{3}{6}} = 2^{\frac{1}{2}} = \sqrt{2}$$

$$\textcircled{7} \quad iX^2 + (-4+3i)X - 3-i = P(X)$$

$$\Delta = (-4+3i)^2 - 4i(-3-i) = \dots = 3-4i = (a+ib)^2 = a^2 - b^2 + i2ab$$

$$|\Delta| \begin{cases} a^2 - b^2 = 3 \\ 2ab = -4 \\ a^2 + b^2 = \sqrt{9+16} = 5 \end{cases}$$

$$L_3 + L_1 : 2a^2 = 8 \Rightarrow a^2 = 4 \Rightarrow a = \pm 2$$

$$L_3 - L_1 : 2b^2 = 2 \Rightarrow b^2 = 1 \Rightarrow b = \pm 1$$

$$" \sqrt{\Delta} " = 2 - i \quad \rightarrow \text{avec } L_2 : a \text{ et } b \text{ ont un signe opposé.}$$

$$\Delta a \text{ racines: } \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-4 \pm 3i) \pm (2-i)}{2i} = \begin{cases} -2-3i \\ -1-i \end{cases}$$

$$P(x) = i(x+2+3i)(x+1+i) = i x^2 + \dots$$

⑨ $xy' + y = \ln(x)$ • HOM: $xy' + y = 0$ $y_H = A e^{-\int \frac{1}{x} dx} = A e^{-\ln x}$

$$y = A e^{\ln(x^{-1})}$$

car $a \ln(b) = \ln(b^a)$

$$y_H = A x^{-1} = \frac{A}{x}$$

• Sol particulière MVC $y_p = \frac{A(x)}{x} = A(x)x^{-1}$

$$y_p' = A'(x)x^{-1} - A(x)x^{-2}$$

$$xy_p' + y_p = A'(x)x - \cancel{A(x)x^{-1}} + \cancel{A(x)x^{-1}} = \ln(x) \Rightarrow A(x) = \int \ln(x) dx$$

$$A = \int 1 \ln x \, dx$$

$$\text{IPP} \quad \left| \begin{array}{ll} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{array} \right.$$

$$A = [x \ln x] - \int \frac{1}{x} \cdot x \, dx = x \ln x - \int 1 \, dx = x \ln x - x = \underline{A(x)}$$

$$y_p = \frac{A(x)}{x} \Rightarrow \boxed{y_p = \ln x - 1}$$

Sols de l'eq complete . $y = \frac{A}{x} + \ln x - 1$

(10) $y' + y = 2e^{-x}$ on veut la sol. p. l'eq. que $f(0) = 3$

Holt . $y = Ae^{-x}$

Sol part : $y_p = ax e^{-x}$ $y_p' = (a - ax)e^{-x}$

$$y_p' + y_p = a e^{-x} = 2e^{-x} \Rightarrow a = 2$$

$$\boxed{y_p = 2x e^{-x}}$$

Sols de l'eq complete : $y = Ae^{-x} + 2x e^{-x}$

$$y(0) = A = 3 \Rightarrow A = 3$$

$$f(x) = (3 + 2x)e^{-x}$$

(11) $y'' + 2y' + y = x$ $f(x) = f'(x) = 0$
 Hott: $c = x^2 + 2x + 1 = (x+1)^2$ $\Delta = 0$ \Rightarrow $\lambda = -1$ (doppelt)

$$y_{\text{Hc}} = (Ax + B)e^{-x} \quad \text{oder } Ax + B \text{ sein}$$

Sol part: $y_{\text{p}} = ax + b$ $y'_{\text{p}} = a$ $y''_{\text{p}} = 0$ $y''_{\text{p}} + 2y'_{\text{p}} + y_{\text{p}} = 2a + ax + b = x$
 $\begin{cases} a = 1 \\ 2a + b = 0 \end{cases} \Rightarrow b = -2$ $y_{\text{p}} = x - 2$

Sol. d. allg. Lsg.: $y = (Ax + B)e^{-x} + x - 2$

$$f(0) = 0 \Rightarrow B - 2 = 0 \Rightarrow \underline{B = 2}$$

$$f'(x) = Ae^{-x} - (Ax + 2)e^{-x} + 1 \Rightarrow f'(0) = A - 2 + 1 = 0 \Rightarrow \underline{A = 1}$$

$$f(x) = (x + 2)e^{-x} + x - 2$$

(12) $y'' - 4y' + 13y = 0$ $\Delta = -36$ \Rightarrow $\lambda_{1,2} = 2 \pm 3i$ $y = e^{2x} (A \cos(3x) + B \sin(3x))$

$$z = x + iy$$

↓

$$N(z) = |z|^2 = x^2 + y^2$$

$$c = e^{i\frac{2\pi}{3}} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\beta = 1 - c$$

$$\textcircled{1} N(\beta) = N\left(1 - \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\right) = N\left(\frac{3}{2} - i\frac{\sqrt{3}}{2}\right)$$

$$= \left(\frac{3}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{9}{4} + \frac{3}{4} = \frac{12}{4} = 3$$

$$\textcircled{2} N(a + cb) = N\left(a + \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)b\right)$$

$$= N\left(a - \frac{1}{2}b + i\frac{\sqrt{3}}{2}b\right)$$

$$= \left(a - \frac{1}{2}b\right)^2 + \left(\frac{\sqrt{3}}{2}b\right)^2 \geq 0$$

$$= \underbrace{a^2}_{\geq 0} - ab + \underbrace{\frac{b^2}{4}}_{\geq 0} + \frac{3b^2}{4} = \underbrace{a^2 - ab + b^2}_{\geq 0} \in \mathbb{Z}$$

car $a, b \in \mathbb{Z}$

donc dans \mathbb{N}

$$\textcircled{3} N(z \cdot z') = N(z) \cdot N(z')$$

$$R_1 e^{i\theta_1} \times R_2 e^{i\theta_2} = R_1 R_2 e^{i(\theta_1 + \theta_2)}$$

$$|z_1 \cdot z_2|^2 = |z_1|^2 |z_2|^2$$

$$N(z_1 z_2) = N(z_1) \cdot N(z_2)$$

$$(14) \quad c^2 = -1 - c \quad ??$$

$$c^2 = \left(e^{i \frac{2\pi}{3}} \right)^2 = e^{i \frac{4\pi}{3}} = -\frac{1}{2} - i \frac{\sqrt{3}}{2}$$

$$-1 - c = -1 - \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = -\frac{1}{2} - i \frac{\sqrt{3}}{2}$$

$$c^2 = -1 - c$$



(16) M. quo si $z \in \mathbb{Z}[i]$ est inversible, alors $N(z) = 1$

Si z est inversible. Il existe z' tel que $z \cdot z' = 1$

$$N(z \cdot z') = N(1) = 1$$

(18) $N(i) = 3 \neq 1$
donc i n'est pas inversible

$$N(z)N(z') = 1 \Rightarrow N(z) = 1$$

(17) $N(3) = 3^2 + 0^2 = 9 \neq 1$, donc 3 n'est pas inversible

$$c \cdot c^2 = e^{i\frac{2\pi}{3}} \cdot e^{i\frac{4\pi}{3}} = e^{i2\pi} = +1 \Rightarrow \text{Cet inversible et } \underline{c^{-1} = c^2}$$

(18) $N(z) = 1$. $z = a + ib$ où a et $b \in \mathbb{Z}$
 $a^2 - ab + b^2$

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

$$e^{ix} = 1 + (ix) + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \dots$$

$$e^{ix} = 1 + \cancel{ix} - \frac{x^2}{2!} - \cancel{i \frac{x^3}{3!}} + \frac{x^4}{4!} + \cancel{i \frac{x^5}{5!}} - \frac{x^6}{6!} + \dots$$

$$e^{-ix} = 1 - \cancel{ix} - \frac{x^2}{2!} + \cancel{i \frac{x^3}{3!}} + \frac{x^4}{4!} - \cancel{i \frac{x^5}{5!}} - \frac{x^6}{6!} + \dots$$

$$\frac{e^{ix} + e^{-ix}}{2} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \mathcal{O}(x^{2n})$$

or

$$+ \mathcal{O}(x^{2n+1})$$

Order 4: $\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \mathcal{O}(x^4)$

Order 5: $\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \mathcal{O}(x^5)$

avec $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$, on obtient :

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \begin{matrix} \mathcal{O}(x^{2n+1}) \\ \text{ou} \\ \mathcal{O}(x^{2n+2}) \end{matrix}$$