



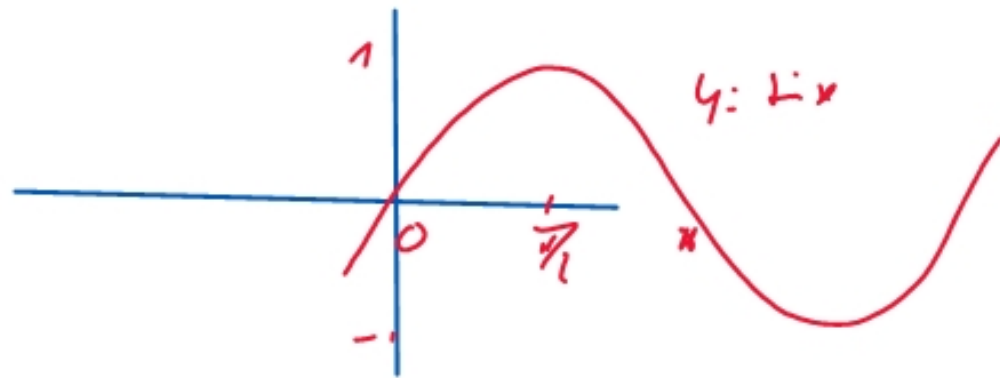
① $\sin(\square - \Delta) = \sin(\square)\cos(\Delta) - \sin(\Delta)\cos(\square)$

② formule du binôme $(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^n \binom{n}{n-k} a^{n-k} b^k$

③ $\cos(2x)$ de 2 façons différentes $\cos(2x) = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

④ $\binom{n}{p} = \frac{n!}{p!(n-p)!}$

⑤ dessin de sin



$$\textcircled{1} \quad 2^2 = 4 \quad -2^3 = -8 \quad (-2)^3 = -8 \quad (-2)^4 = 16, \quad (-2^4) = -16$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}; \quad (-2)^{-2} = \frac{1}{4}$$

$$\textcircled{2} \quad A = -2 \quad A^{-10} = (-2)^{-10} = \frac{1}{(-2)^{10}} = \frac{1}{2^{10}} = \frac{1}{1024}$$

$$\frac{1}{4} A^{-4} = \frac{1}{4} (-2)^{-4} = \frac{1}{4 \times (-2)^4} = \frac{1}{2^2 \cdot 2^4} = \frac{1}{2^6} = \frac{1}{64}$$

$$\textcircled{3} \quad 2^8 \times 2^{12} = 2^{20}; \quad (6 \times 25)^2 \times 5^2 = (2 \times 3 \times 5^2)^2 \times 5^2$$

$$= 2^2 \times 3^2 \times 5^4 \times 5^2 = 2^2 \times 3^2 \times 5^6$$

$$\textcircled{4} \quad (6 \times 25)^2 \times 12^4 = (2 \times 3 \times 5^2)^2 \times (2^2 \times 3)^4$$

$$= (2^2 \times 3^2 \times 5^4) \times 2^8 \times 3^4 = 2^{10} \times 3^6 \times 5$$

$$\textcircled{4} \quad \frac{10^2 \times 45}{(12 \times 20)^2} = \frac{(2 \times 5)^2 \times 3^2 \times 5}{(2^2 \times 3 \times 2^2 \times 5)^2} = \frac{2^2 \times 5^2 \times 3^2 \times 5}{(2^4 \times 3 \times 5)^2} = \frac{2^2 \times 3 \times 5^3}{2^8 \times 3^2 \times 5^2}$$

$$\frac{5^3}{5^2} = \frac{5^2 \times 5}{5^2} \quad \frac{2^2}{2^8} = \frac{2^2}{2^2 \times 2^6} = \frac{5}{2^6}$$

$$\textcircled{5} \quad c = 0,001 \text{ i.e., } 001 = 10^{-4} \times 10^{-3} = 10^{-7}$$

$$d = \frac{1000}{0,00001} = \frac{10^3}{10^{-5}} = 10^3 \times 10^5 = 10^8$$

$$\textcircled{6} \quad 3,4 \times 10^4 + 2,5 \times 10^5 = 10^4 \left(3,4 + \frac{2,5 \times 10}{2,5} \right) = 10^4 \times 28,4$$

$$10^4 + 10^{-2} = 10^3 (10 + 10^{-5})$$

$$\begin{aligned}
 \textcircled{7} \quad 35^2 \times 20 \times 14 &= (5 \times 7)^2 \times (2^2 \times 5) \times (2 \times 7) \\
 &= 5^2 \times 7^2 \times 2^2 \times 5 \times 2 \times 7 = 2^3 \times 5^3 \times 7^3 = (2 \times 5 \times 7)^3 \\
 &= 70^3
 \end{aligned}$$

$$\begin{array}{r|l}
 2268 & 2 \\
 \hline
 1134 & 2 \\
 \hline
 567 & \leftarrow 3 \\
 189 & \leftarrow 3 \\
 63 & \leftarrow 3 \\
 \hline
 21 & \leftarrow 3 \\
 7 & 7
 \end{array}$$

$$2268 = 2^2 \times 3^4 \times 7$$

$$\textcircled{9} \quad \frac{1}{x^{-1/2}} = x^{1/2}$$

$$x^2 \times x^{-1/2} = x^{2-1/2} = x^{4/2-1} = x^{3/2}$$

$$\frac{x^{-1/2}}{x} = x^{-1/2-1} = x^{-3/2}$$

(III Racines Carrées)

Applications aux polynômes de degré 2: $ax^2 + bx + c$

$$\Delta = b^2 - 4ac \quad \text{racine} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

ex: Trouver les 2 racines de $P(x) = i x^2 - 5x + 1 - 7i$, puis factoriser \mathcal{L} .

(Rappel: quand a est racine de \mathcal{L} (càd $\mathcal{L}(a) = 0$), on peut factoriser par $x - a$)

$$\Delta = (-5)^2 - 4 \cdot i \cdot (1 - 7i) = 25 - 4i + 28i^2 = 25 - 4i - 28 = -3 - 4i$$

on calcule $\sqrt{\Delta} = \sqrt{-3 - 4i} = a + ib$

$$(a + ib)^2 = \Delta$$

$$P(x) = \underline{\underline{i}}x^2 - 5x + 1 - 7i$$

$$\Delta = -3 - 4i = (a+ib)^2 = a^2 - b^2 + i2ab$$

avec $|\Delta|$
$$\begin{cases} a^2 - b^2 = -3 \\ 2ab = -4 \\ a^2 + b^2 = \sqrt{(-3)^2 + (-4)^2} = 5 \end{cases}$$

$$L_1 + L_3: 2a^2 = 2 \Rightarrow a^2 = 1 \Rightarrow \underline{a = \pm 1}$$

$$L_3 - L_1: 2b^2 = 8 \Rightarrow b^2 = 4 \Rightarrow \underline{b = \pm 2}$$

avec L_2 , on voit que a et b ont un signe opposé, donc $\sqrt{\Delta} = \pm(1 - 2i)$

les racines de Δ sont: $\frac{5 \pm (1 - 2i)}{2i} = \begin{cases} \frac{6 - 2i}{2i} = \frac{2(3 - i)}{2i} \\ \frac{4 + 2i}{2i} = \frac{2(2 + i)}{2i} \end{cases}$

$$\begin{cases} \frac{(3 - i)xi}{ixi} = \frac{3i + 1}{-1} = \underline{-1 - 3i} \\ \frac{(2 + i)xi}{ixi} = \frac{2i - 1}{-1} = \underline{1 - 2i} \end{cases} \text{ les 2 racines}$$

$$P(x) = \underline{\underline{i}}(x + 1 + 3i)(x - 1 + 2i)$$

ex 2. $P(x) = x^2 - (1-2i)x + 1-7i$ Factoriser \mathbb{C}

$$\Delta = (1-2i)^2 - 4(1-7i) = 1 - 4i - 4 - 4 + 28i = -7 + 24i$$

$$\left\{ \begin{array}{l} a^2 - b^2 = -7 \\ 2ab = 24 \end{array} \right. \quad \begin{array}{l} = (a+ib)^2 \\ = a^2 - b^2 + i2ab \end{array}$$

1) $a^2 + b^2 = \sqrt{49 + 576} = \sqrt{625} = 25$

$$L_3 + L_1: 2a^2 = 18 \Rightarrow a^2 = 9 \quad a = \pm 3$$

$$L_3 - L_1: 2b^2 = 32 \Rightarrow b^2 = 16 \quad b = \pm 4$$

$$L_2: \hat{m} \text{ signe } \sqrt{\Delta} = \pm (3+4i)$$

$$2 \text{ racines: } \frac{1-2i \pm (3+4i)}{2} = \begin{cases} \frac{4+2i}{2} = 2+i \\ \frac{-2-6i}{2} = -1-3i \end{cases}$$

$$\underline{P(x) = (x - 2 - i)(x + 1 + 3i)}$$

IV Racines $n^{\text{ième}}$ d'un complexe

On cherche les complexes $z = re^{i\theta}$ tels que $z^n = Re^{i\varphi}$

$$(re^{i\theta})^n = Re^{i\varphi} \Leftrightarrow r^n e^{in\theta} = Re^{i\varphi}$$

$$\text{donc } r^n = R \Rightarrow r = \sqrt[n]{R} = \underline{\underline{R^{1/n}}}$$

$$n\theta = \varphi + 2k\pi \Rightarrow \theta = \frac{\varphi}{n} + \frac{2k\pi}{n}$$

□ Les racines $n^{\text{ièmes}}$ de $Re^{i\varphi}$ sont les $z_k = R^{1/n} e^{i\left(\frac{\varphi}{n} + \frac{2k\pi}{n}\right)}$
où $k = 0, 1, \dots, n-1$

ex 1 : Quelles sont les racines cubiques de i ?

$$i = e^{i\pi/2}$$

Les racines cubiques de i sont les $z_k = 1^{1/3} e^{i\left(\frac{\pi/2}{3} + \frac{2k\pi}{3}\right)}$ où $k = 0, 1$
ou 2

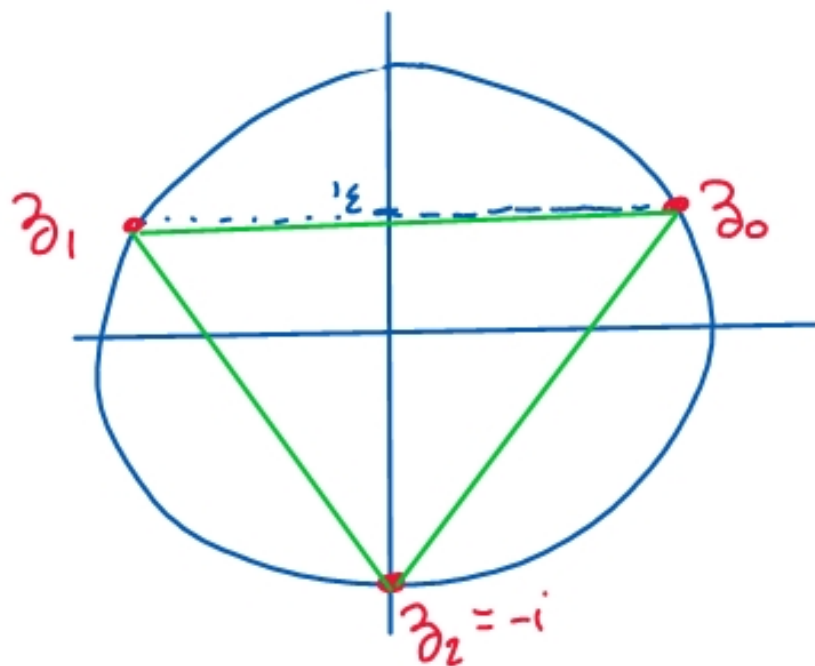
$$z_k = e^{i\left(\frac{\pi}{6} + \frac{2k\pi}{3}\right)}$$



pour $k=0$: $z_0 = e^{i\frac{\pi}{6}}$

pour $k=1$: $z_1 = e^{i(\frac{\pi}{6} + \frac{2\pi}{3})} = e^{i\frac{\pi+4\pi}{6}} = e^{i\frac{5\pi}{6}}$

pour $k=2$: $z_2 = e^{i(\frac{\pi}{6} + \frac{4\pi}{3})} = e^{i\frac{9\pi}{6}} = e^{i\frac{3\pi}{2}} = -i$



$$(z_0)^3 = (e^{i\frac{\pi}{6}})^3 = e^{i\frac{3\pi}{6}} = e^{i\frac{\pi}{2}} = i$$

$$(z_1)^3 = (e^{i\frac{5\pi}{6}})^3 = e^{i\frac{15\pi}{6}} = e^{i\frac{5\pi}{2}} = e^{i\frac{\pi}{2}} = i$$

$$(z_2)^3 = (e^{i\frac{3\pi}{2}})^3 = e^{i\frac{9\pi}{2}} = e^{i\frac{\pi}{2}} = i$$

en fait les racines $n^{\text{ièmes}}$ forment un polygone régulier à n côtés

ex 2: Trouver les racines 5^{ième} de $32 = z = 32e^{i0}$

les racines 5^{ième} de 32 sont les $z_k = 32^{1/5} e^{i(0 + \frac{2k\pi}{5})}$ où $k=0,1,\dots,4$

$$32 = 2^5 \Rightarrow 32^{1/5} = 2$$

$$k=0 \quad z_0 = 2$$

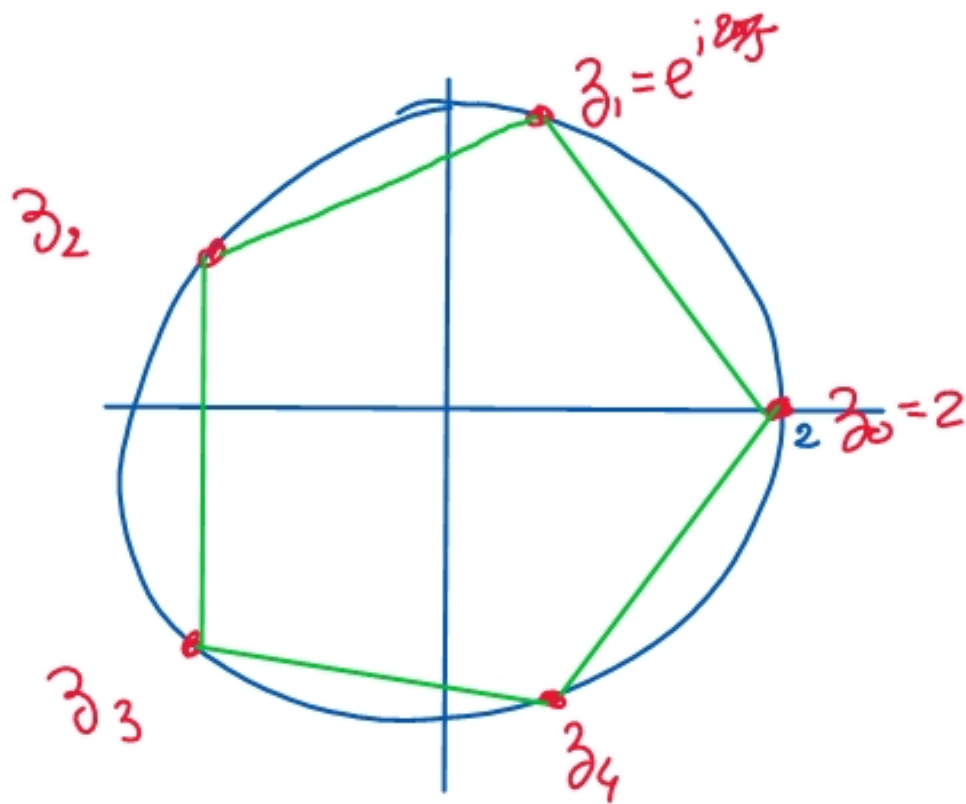
$$z_1 = 2e^{i2\pi/5}$$

$$z_2 = 2e^{i4\pi/5}$$

$$z_3 = 2e^{i6\pi/5}$$

$$z_4 = 2e^{i8\pi/5}$$

$$z_5 = 2e^{i10\pi/5} = 2e^{i2\pi} = z_0$$



□ la somme des racines $n^{\text{ième}}$ d'un complexe vaut 0.

□ les racines $n^{\text{ième}}$ de $Re^{i\theta}$ sont les $z_k = R^{\frac{1}{n}} e^{i(\frac{\theta}{n} + \frac{2k\pi}{n})}$
où $k=0, 1, \dots, n-1$

$$\begin{aligned} \sum_{k=0}^{n-1} z_k &= \sum_{k=0}^{n-1} R^{\frac{1}{n}} e^{i\frac{\theta}{n}} \times e^{i\frac{2k\pi}{n}} \\ &= R^{\frac{1}{n}} e^{i\frac{\theta}{n}} \left(\sum_{k=0}^{n-1} e^{i\frac{2k\pi}{n}} \right) = R^{\frac{1}{n}} e^{i\frac{\theta}{n}} \left(\sum_{k=0}^{n-1} \left(e^{i\frac{2\pi}{n}} \right)^k \right) \end{aligned}$$

$$\sum_{k=0}^{n-1} r^k = 1 + r + r^2 + \dots + r^{n-1} = \frac{1-r^n}{1-r}$$

$$= \frac{1 - \left(e^{i\frac{2\pi}{n}} \right)^n}{1 - e^{i\frac{2\pi}{n}}}$$

$$= \frac{1 - e^{i2\pi}}{1 - e^{i\frac{2\pi}{n}}} = \frac{1-1}{1 - e^{i\frac{2\pi}{n}}} = 0$$

$$\sum_{k=0}^{n-1} r^k$$

où $r = e^{i\frac{2\pi}{n}}$

□