

penso 2

$$2^7 + 2^8 + \dots + 2^{36} = 2^7 (1 + 2 + 2^2 + \dots + 2^{29})$$

$$= 2^7 \frac{2^{30} - 1}{2 - 1} = 2^{37} - 2^7$$

$$\cos(7x) + \dots + \cos(36x) = \operatorname{Re} \left(e^{i7x} + \dots + e^{i36x} \right)$$

$$= \operatorname{Re} e^{i7x} \left[1 + e^{i7x} + (e^{i7x})^2 + \dots + (e^{i7x})^{29} \right]$$

$$= \operatorname{Re} \left(e^{i7x} \frac{e^{i30x} - 1}{e^{i7x} - 1} \right) = \operatorname{Re} \left(e^{i7x} \frac{e^{i15x} (e^{i15x} - e^{-i15x})}{e^{i7x/2} - e^{-i7x/2}} \right)$$

$$= \operatorname{Re} \left(e^{i(7+15-\frac{1}{2})x} \frac{e^{i15x} \sin(15x)}{2i \sin(\frac{x}{2})} \right)$$

$$= \operatorname{Re} \left(e^{i \frac{43}{2} x} \frac{\sin(15x)}{\sin(\frac{x}{2})} \right)$$

$$= \cos\left(\frac{43}{2}x\right) + i \sin\left(\frac{43}{2}x\right)$$

$$\rightarrow \cos\left(\frac{43}{2}x\right) \frac{\sin(15x)}{\sin(\frac{x}{2})}$$