

$$\sum_{k=0}^{n-1} (-1)^k t^k = 1 - t + t^2 - t^3 + \dots + (-1)^{n-1} t^{n-1}$$

$$\begin{aligned}
 &= \sum_{k=0}^{n-1} (-t)^k \\
 &= \frac{1 - (-t)^n}{1 - (-t)} \\
 &= \frac{1 - (-t)^n}{1+t}
 \end{aligned}$$

$$\sum_{k=0}^{n-1} (-1)^k t^k = \frac{1}{1+t} - \frac{(-t)^n}{1+t}$$

$$\Rightarrow \frac{1}{1+t} = \sum_{k=0}^{n-1} (-1)^k t^k + \frac{(-t)^n}{1+t}$$