

$$\sum_n = \sum_{k=0}^{2n+1} e^{i \frac{kr\pi}{n+1}} = \sum_{k=0}^{2n+1} \left(e^{i \frac{r\pi}{n+1}} \right)^k$$

→ somme partielle géom de raison $e^{i \frac{r\pi}{n+1}}$

$$\sum_n = \begin{cases} e^{i \frac{r\pi}{n+1}} = 1 \text{ lorsque } \frac{r\pi}{n+1} = 2 \times k \Rightarrow r = 2k(n+1) \\ e^{i \frac{r\pi}{n+1}} (2n+2) \frac{1 - e^{-i \frac{r\pi}{n+1}}}{1 - e^{-i \frac{r\pi}{n+1}}} \text{ qd } r \neq 2k(n+1) \\ \underbrace{1 + 1 + \dots + 1}_{2n+2 \text{ fois}} = 2n+2 \text{ qd } r = 2k(n+1) \end{cases}$$

$$= \begin{cases} \frac{e^{i 2\pi} - 1}{e^{i \frac{r\pi}{n+1}} - 1} = \frac{0}{\dots} = 0 \\ \text{ou} \\ 2n+2 \text{ qd } r = 2k(n+1) \end{cases}$$