

$$x_k = (2\cos\alpha) x_{k-1} - x_{k-2}, \quad \forall k \geq 2$$

avec $x_k = r^k$:

$$r^k = (2\cos\alpha) r^{k-1} - r^{k-2}, \quad \forall k \geq 2$$

Si $r \neq 0$, on peut simplifier par r^{k-2} :

$$r^2 - 2\cos\alpha r - 1 = 0 \quad (\text{car } r^2 - 2\cos\alpha r + 1 = 0)$$

$$\Delta = 4\cos^2\alpha - 4 = 4(\cos^2\alpha - 1) = -4\sin^2\alpha$$
$$= (2i\sin\alpha)^2$$

$$r = \frac{+2\cos\alpha \pm 2i\sin\alpha}{2}$$

$$= \cos\alpha \pm i\sin\alpha = e^{\pm i\alpha}$$