

$$f(x) = \sum_0^{\infty} \frac{(-1)^n x^{2n}}{2n+1}$$

et

$$f(x) = \sum_0^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

↳ n pairs

$$+ \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{(2n+1)!} x^{2n+1}$$

↳ impairs

$$= \sum_0^{\infty} \frac{(-1)^n x^{2n}}{2n+1}$$

+

$$0$$

Donc

$$\frac{f^{(2n)}(0)}{(2n)!} = \frac{(-1)^n}{2n+1}$$

et

$$f^{(2n+1)}(0) = 0$$

$$f^{(2n)}(0) = \frac{(-1)^n (2n)!}{2n+1}$$