

Total test 6

①

reg'ky = arctan

Met: $y \Rightarrow A e^{-\int \frac{1}{x} dx} = A e^{-\ln x} = \frac{A}{x} = y_H(x)$

Sol part: $y_H = \frac{A(x)}{x}$ $y_H' = \frac{A'(x)x - A(x)}{x^2}$

reg'ky = $\frac{A'(x)x - A(x)}{x} + \frac{A(x)}{x} = A'(x) = \arctan x$

$A(x) = \int \arctan x dx$ $\int \frac{1}{x} dx = \ln|x|$
 $u = \arctan x$ $u' = \frac{1}{1+x^2}$
 $v' = 1$ $v = x$

$A(x) = [x \arctan x] - \int \frac{x}{1+x^2} dx = x \arctan x - \frac{1}{2} \ln(1+x^2)$

$y_H(x) = \arctan x - \frac{1}{2x} \ln(1+x^2)$

Sols de l'eq complete: On ajoute le deux

2. On pose $Z = z^3$ $Z^2 - Z + 1 = 0$ $\Delta = -3 = (\sqrt{3}i)^2$

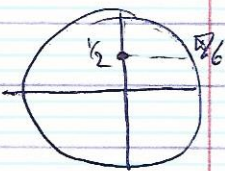
$Z = \frac{1 \pm \sqrt{3}i}{2} = e^{\pm i\pi/3}$



On checke alors les 3 racines cubiques de $e^{i\pi/3}$ et $e^{-i\pi/3}$

les 3 racines de $e^{i\pi/3}$ sont $e^{i(\pi/9 + 2k\pi/3)}$ ou $k=0,1,2$
 $e^{-i\pi/3}$ ont $k=0,1,2$

3. $\frac{\sqrt{3}}{2} + \frac{i}{2} = e^{i\pi/6}$ les deux racines complexes sont $\pm e^{i\pi/12}$



deuxieme methode: $\frac{\sqrt{3}}{2} + \frac{i}{2} = (a+ib)^2 = a^2 - b^2 + i2ab$

$a^2 - b^2 = \frac{\sqrt{3}}{2}$ $\Rightarrow 2a^2 = 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}$ $a = \pm \frac{\sqrt{2+\sqrt{3}}}{2}$

$2ab = \frac{1}{2}$

module: $a^2 + b^2 = 1$ et $b = \frac{1}{4a} = \pm \frac{1}{2\sqrt{2+\sqrt{3}}}$

les deux racines sont $\pm \left(\frac{\sqrt{2+\sqrt{3}}}{2} + i \frac{1}{2\sqrt{2+\sqrt{3}}} \right) = \pm \left(\cos\left(\frac{\pi}{12}\right) + i \sin\left(\frac{\pi}{12}\right) \right)$

Donc $\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{2+\sqrt{3}}}{2}$