

Oral 2017 Exo 5

① La justification est hors programme -
On peut supposer que l'étude a été guidée

②
$$I = \int \ln x \, dx = [x \ln x] - \int 1 \, dx$$

$$= x \ln x - x$$

$$\text{Euler} \left| \begin{array}{l} u = \ln x \quad u \leq \frac{1}{2} \\ u' = 1 \quad u = x \end{array} \right.$$

$$\int_a^1 x \ln x \, dx = [x \ln x - x]_a^1 = -1 - a \ln a + a$$

Or $\lim_{a \rightarrow 0} a \ln a = 0$ (∇ des courbes asymptotes)

donc $\int_0^1 \ln x \, dx \in \mathbb{R}$ (et vaut -1).

$$I = \int_0^{\pi/2} \ln(\sin x) \, dx$$

$$\int_0^{\pi/2} \ln(\sin x) \, dx \in \mathbb{R}$$

$$\int_0^{\pi/2} \ln x \, dx \in \mathbb{R}$$

$$\text{donc } I \in \mathbb{R} \text{ (et } \nabla \text{ de } \mathbb{R})$$

$$J = \int_0^{\pi/2} \ln(\cos x) \, dx$$

$$\text{chgt } X = \frac{\pi}{2} - x \quad dX = -dx$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x = \cos X$$

$$\text{(et } \sin\left(\frac{\pi}{2} - x\right) = \cos x = \sin X \text{)}$$

$$J = \int_{x=\pi/2}^{x=0} \ln(\sin X) (-dX) = \int_0^{\pi/2} \ln(\sin X) \, dX$$

$$I = J$$