

Travail 2017 Exo 4

2.

$$xy' - 2y = \ln x$$

Méthode : $y_H = A e^{\int \frac{2}{x} dx} = A e^{2 \ln x} = \boxed{A x^2 = y_H}$

Sol part. p.V.C $y_P = A(x) x^2$ $y'_P = A'(x) x^2 + 2A(x) x$

$$x y'_P - 2 y_P = A'(x) x^3 + 2A(x) x^2 - 2A(x) x^2 = \ln x$$

$$A'(x) = \int \frac{\ln x}{x^3} dx \quad \text{Chgt de variable } X = \ln x \quad x = e^X \quad dx = e^X dx$$

$$A'(x) = \int X e^{-3X} e^X dx = \int X e^{-2X} dx$$

Essai $\left| \begin{array}{l} u = X \quad u' = 1 \\ v' = e^{-2X} \quad v = \frac{e^{-2X}}{-2} \end{array} \right.$

$$A'(x) = \left[X \frac{e^{-2X}}{-2} \right] + \frac{1}{2} \int e^{-2X} dx = -\frac{X e^{-2X}}{2} - \frac{1}{4} e^{-2X}$$

$$A(x) = -\frac{\ln x}{2x^2} + \frac{1}{4x^2} = \frac{-1 - 2 \ln x}{4x^2}$$

$$\boxed{y_P(x) = \frac{-1 - 2 \ln x}{4}}$$

Sols de l'eq complète : $y = A x^2 + \frac{-1 - 2 \ln x}{4}$ ou $A(x) x^2$

3. $\det \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} = \begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix} = (-1) \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = 4 \neq 0$

$\det(A) \neq 0$, donc A est inversible

A est diagonalisable car symétrique