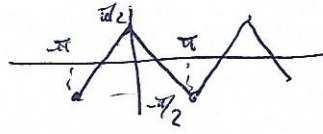


Cher 17 exo 2

① $f(x) = \begin{cases} \frac{\pi}{2} + x, & \forall x \in [-\pi; 0[\\ \frac{\pi}{2} - x \sin(\cos(x)) \end{cases}$



est paire et
2π - période de f

$b_n = 0, \forall n \geq 1$

$a_0 = \frac{2}{2\pi} \int_0^\pi (\frac{\pi}{2} - x) dx = 0 \quad \boxed{a_0 = 0}$

$a_n = \frac{4}{2\pi} \int_0^\pi (\pi - x) \cos(nx) dx$ E.P.P $\begin{cases} u = \pi - x & u' = -1 \\ v' = \cos(nx) & v = \frac{\sin(nx)}{n} \end{cases}$

$a_n = \frac{2}{\pi} \left[(\pi - x) \frac{\sin(nx)}{n} \right]_0^\pi + \frac{2}{n\pi} \int_0^\pi \sin(nx) dx = 0 - \frac{2}{n^2\pi} \left[\cos(nx) \right]_0^\pi$

$a_n = \frac{2(1 - \cos(n\pi))}{n^2\pi} = \frac{2(1 - (-1)^n)}{n^2\pi} = \begin{cases} \frac{4}{n^2\pi} & \text{qd } n \text{ est impair} \\ 0 & \text{qd } n \text{ est pair} \end{cases}$

$f(x) = \sum_{n \text{ impair}} \frac{4}{n^2\pi} \cos(nx) = \frac{4}{\pi} \sum_{p=0}^{+\infty} \frac{1}{(2p+1)^2} \cos((2p+1)x)$

② $\frac{1}{z^2} + z^{2H} = 0 \Leftrightarrow z^4 + z^2 + 1 = 0 \Leftrightarrow \frac{z^6 - 1}{z^2 - 1} = 0$

$z^6 = 1 \Leftrightarrow z_k = e^{i \frac{2k\pi}{6}}$

$z^6 - 1 = (z - 1)(z - e^{i\pi/3})(z - e^{i2\pi/3})(z + 1)(z - e^{-i\pi/3})(z - e^{-i2\pi/3})$
 $z^2 - 1 = (z - 1)(z + 1)$

Donc $\frac{z^6 - 1}{z^2 - 1} = (z - e^{i\pi/3})(z - e^{i2\pi/3})(z - e^{-i2\pi/3})(z - e^{-i\pi/3})$

il y a 4 solutions: $e^{\pm i\pi/3}$ et $e^{\pm i2\pi/3}$