

2022 - TH161 Rouen.

① Résoudre $\sin^3 x + \cos^3 x = 1$

On étudie la fonction $x \mapsto f(x) = \sin^3 x + \cos^3 x$. f est 2π -périodique

$$f'(x) = 3\sin^2 x \cos x - 3\cos^2 x \sin x = 3\cos x \sin x (\sin x - \cos x)$$

$$f'(x) = 0 \Leftrightarrow \begin{cases} \cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi \\ \sin x = 0 \Leftrightarrow x = k\pi \\ \cos x = \sin x \Leftrightarrow x = \frac{\pi}{4} + k\pi \end{cases}$$

| | | | | | | | | |
|---------|---|----------------------|-----------------|-------|-----------------------|------------------|--------|---|
| | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | π | $\frac{3\pi}{4}$ | $\frac{3\pi}{2}$ | 2π | |
| $f'(x)$ | 0 | - | + | - | + | - | + | 0 |
| $f(x)$ | 1 | $\frac{1}{\sqrt{2}}$ | 1 | -1 | $-\frac{1}{\sqrt{2}}$ | -1 | 1 | |

$$f\left(\frac{\pi}{8}\right) = 3 \cdot \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \left(\frac{1}{2} - \frac{\sqrt{2}}{2}\right) < 0$$

$$f\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}\right)^3 + \left(\frac{\sqrt{2}}{2}\right)^3 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Ainsi : $S = \left\{ \frac{k\pi}{2} / k \in \mathbb{Z} \right\} \cup \left\{ \frac{\pi}{2} + k\pi / k \in \mathbb{Z} \right\}$

② $z^2 - (1-2i)z + 1-7i = 0$

$$\Delta = (1-2i)^2 - 4(1-7i) = -7 + 24i = (a+ib)^2 = a^2 - b^2 + i 2ab$$

$$| \Delta | \begin{cases} a^2 - b^2 = -7 \\ 2ab = 24 \\ a^2 + b^2 = \sqrt{49 + 576} = \sqrt{625} = 25 \end{cases}$$

$$L_1 + L_2 : 2a^2 = 18 \Rightarrow a^2 = 9 \quad a = \pm 3$$

$$L_2 : ab = 12 \Rightarrow b = \pm 4$$

$$\Delta = (3 + 4i)^2$$

les deux racines sont :

$$z = \frac{1-2i \pm (3+4i)}{2} = \begin{cases} \frac{4+2i}{2} = 2+i \\ \frac{-2-6i}{2} = -1-3i \end{cases}$$