

① $\mathcal{L}(x) = (1+i)x^2 + 2x - 3 + 11i$

$$\Delta = 4 - 4(1+i)(-3+11i) = 4 - 4(-3+11i-3i-11) = 4 - 4(-14+8i) = 4+56-32i = 60-32i = 4(15-8i)$$

On cherche les racines carrées de $\delta = 15-8i = (a+ib)^2 = a^2-b^2+i2ab$

$$|A| \begin{cases} a^2-b^2 = 15 \\ 2ab = -8 \\ a^2+b^2 = \sqrt{225+64} = \sqrt{289} = 17 \end{cases}$$

$$l_1+l_2: 2a^2 = 32 \Rightarrow a^2 = 16 \Rightarrow a = \pm 4$$

$$l_2: ab = -4 \Rightarrow b = \mp 1$$

$$\delta = (4-i)^2$$

$$\Delta = 4(4-i)^2 = (8-2i)^2$$

les deux racines de \mathcal{L} sont : $\frac{-2 \pm (8-2i)}{2+2i} = \frac{6-2i}{4+2i} = \frac{3-i}{2+i} = \frac{(3-i)(1-i)}{2} = \frac{2-4i}{2} = (1-2i)$
 $\frac{-10+2i}{2+2i} = \frac{-5+i}{1+i} = \frac{(-5+i)(1-i)}{2} = \frac{-6-4i}{2} = (-3-2i)$

donc $\mathcal{L}(x) = (1+i)[x - 1+2i][x + 3+2i]$

② Hom: $y_H = Ae^{-x}$

Sol part pour x : $y_{H1} = ax+b$. On trouve $y_{H1} = x-1$

Sol part pour e^x : $y_{H2} = ae^x$. On trouve $y_{H2} = -\frac{1}{2}e^x$

Sol part pour $\cos x$: $y_{H3} = a \cos x + b \sin x$ $y_{H3}' = -a \sin x + b \cos x$

$$y_{H3}' + y_{H3} = (a+b) \cos x + (b-a) \sin x = \cos x$$

$$\begin{cases} a+b=1 \\ b-a=0 \end{cases} \quad \begin{matrix} l_1+l_2: 2b=1 & b=\frac{1}{2} \\ l_1-l_2: 2a=1 & a=\frac{1}{2} \end{matrix}$$

$$y_{H3} = \frac{1}{2} \cos x + \frac{1}{2} \sin x$$

Sol_s de l'eq complète:

$$y = Ae^{-x} + x - 1 + (-\frac{1}{2}e^x) + \frac{1}{2} \cos x + \frac{1}{2} \sin x \quad \text{où } A \in \mathbb{R}$$