

① $\det_{\text{Euc}}(\vec{a}, \vec{b}, \vec{c}) = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} \stackrel{\text{dévelop}}{=} 1 \cdot \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 \cdot (1-1) = -1 \neq 0$, donc B_1 est une base de \mathbb{R}^3

$\det_{\text{Euc}}(\vec{i}, \vec{j}) = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1-2 = -1$, donc B_2 est une base de \mathbb{R}^2

$M_{B_2}^B f = \begin{pmatrix} \text{Coord de } f(\vec{a}) \\ \text{des } B_2 \end{pmatrix} = \begin{pmatrix} f(\vec{a}) \\ f(\vec{b}) \\ f(\vec{c}) \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 5 & 0 \\ 2 & 5 & 1 \end{pmatrix}$

$f(\vec{a}) = (0, 2) = 2(1, 2) - 2(1, 1) = 2\vec{i} - 2\vec{j} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}_{B_2}$

$f(\vec{b}) = (0, 5) = 5\vec{i} - 5\vec{j} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}_{B_2}$

$f(\vec{c}) = (1, 1) = \vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{B_2}$

② Holm: $y'' + my' + ny = 0$ $\Delta(x) = x^2 + mx + 1$ (une racine double -1)

$\Delta = m^2 - 4$

1^{er} cas: $m = 2$

$y_H = (Ax + B)e^{-x}$

2^{er} cas: $m = -2$

$y_H = (Ax + B)e^x$

3^{er} cas: $m \in]-2; 2[\quad \Delta < 0$

racines: $-\frac{m}{2} \pm i\sqrt{\frac{4-m^2}{4}}$

$y_H = \left(A \cos \sqrt{\frac{4-m^2}{4}} x + B \sin \sqrt{\frac{4-m^2}{4}} x \right) e^{-\frac{m}{2}x}$

4^{er} cas: $m \in]0; -2[\cup]2; +\infty[$

racines: $-\frac{m}{2} \pm \sqrt{\frac{m^2-4}{4}}$

$y_H = A e^{-\frac{m+\sqrt{m^2-4}}{2}x} + B e^{-\frac{m-\sqrt{m^2-4}}{2}x}$

Sol particulière: $y_p = ax + b \quad y_p' = a \quad y_p'' = 0$

$y_p'' + my_p' + y_p = ma + ax + b = x \Rightarrow a=1 \text{ et } b=-m$

$y_p = x - m$

Sol de l'équation complète: On ajoute les deux.