

ATS 2021 (MATH Arnaud)

$$\textcircled{1} \quad \frac{x}{(x+1)(x+2)(x+3)} \underset{x \rightarrow +\infty}{\sim} \frac{x}{x^3} = \frac{1}{x^2}$$

$$\int_{+\infty}^{+\infty} \frac{1}{x^2} dx \text{ CU (Riemann)}, \text{ donc } \int_{(x+1)(x+2)(x+3)}^{+\infty} \frac{x}{x} dx \text{ CU (TdsV)}$$

$$\frac{x}{(x+1)(x+2)(x+3)} = \frac{a}{x+1} + \frac{b}{x+2} + \frac{c}{x+3}$$

$\frac{a}{x+1} = 0$ car le degré du numérateur est $<$ au degré du dénominateur

$$a = \frac{x}{(x+1)(x+3)} \Big|_{x=-1} = \frac{-1}{2} \quad b = \frac{x}{(x+1)(x+3)} \Big|_{x=-2} = \frac{-2}{-1 \cdot 1} = 2$$

$$c = \frac{x}{(x+1)(x+2)} \Big|_{x=-3} = \frac{-3}{(-2)(-1)} = -\frac{3}{2}$$

$$I = \int_0^{+\infty} \left(\frac{-1/2}{x+1} + \frac{2}{x+2} - \frac{3/2}{x+3} \right) dx. \text{ On calcule } I(X) = \int_0^X (\dots) dx$$

$$I(X) = -\frac{1}{2} \left[\ln(x+1) \right]_0^X + 2 \left[\ln(x+2) \right]_0^X - \frac{3}{2} \left[\ln(x+3) \right]_0^X$$

$$= -\frac{1}{2} \ln(X+1) + 2 \ln(X+2) - 2 \ln 2 - \frac{3}{2} \ln(X+3) + \frac{3}{2} \ln(3)$$

$$= \ln \left(\frac{(X+2)^2}{(X+1)^{1/2} (X+3)^{3/2}} \right) - 2 \ln 2 + \frac{3}{2} \ln(3)$$

$$\text{On fait tendre } X \text{ vers } +\infty: \frac{(X+2)^2}{(X+1)^{1/2} (X+3)^{3/2}} \underset{x \rightarrow +\infty}{\sim} \frac{x^2}{x^{1/2} x^{3/2}} = 1 \rightarrow 1$$

et $\ln(1) = 0$, donc

$$I = -2 \ln 2 + \frac{3}{2} \ln 3$$

$$\textcircled{2} \quad z^{2n} + z^n + 1 = 0 \quad \text{On pose } z = z^n$$

$$z^2 + z + 1 = 0 \quad \Delta = -3 = (i\sqrt{3})^2$$

$$z = \frac{-1 \pm i\sqrt{3}}{2} = e^{\pm i\frac{2\pi}{3}}$$

$$\text{1}^{\text{er}} \text{ cas: } z^n = e^{i\frac{2\pi k}{3}}$$

$$z_k = e^{i\left(\frac{2\pi k}{3n} + \frac{2\pi h}{n}\right)} \quad \text{pour } k=0, 1, \dots, n-1$$

$$\text{2}^{\text{e}} \text{ cas: } z^n = e^{-i\frac{2\pi k}{3}}$$

$$z_k = e^{-i\left(\frac{2\pi k}{3n} + \frac{2\pi h}{n}\right)} \quad \text{pour } k=0, 1, \dots, n-1$$