

ATS 2019 (THAVEL Florin)

$$\textcircled{1} z^3 + 3z - 2i = 0$$

avec $z=i$: $i^3 + 3i - 2i = -i + 3i - 2i = 0$ est racine

$$\begin{array}{r} z^3 + 3z - 2i \\ -(z^3 - iz^2) \\ \hline +iz^2 + 3z - 2i \\ -(iz^2 + z) \\ \hline 2z - 2i \\ -(2z - 2i) \\ \hline 0 \end{array}$$

$$\frac{z^3 + 3z - 2i}{z - i} = z^2 + iz + 2$$

$$\Delta = i^2 - 8 = -9 = (3i)^2$$

$$z = \frac{-i \pm 3i}{2} = \begin{cases} i \\ -2i \end{cases}$$

Il y a 2 racines :

i est racine double
et $-2i$ racine simple

$$\textcircled{2} \frac{1}{x^2 - 3x + 2} = \frac{1}{(x-2)(x-1)} = \frac{a}{x-2} + \frac{b}{x-1} = \frac{1}{x-2} + \frac{1}{x-1}$$

$$a = \frac{1}{x-1} \Big|_{x=2} = 1 \quad b = \frac{1}{x-2} \Big|_{x=1} = -1$$

$$S_N = \sum_{n=3}^N \frac{1}{n^2 - 3n + 2} = \sum_{n=3}^N \frac{1}{n-2} - \sum_{n=3}^N \frac{1}{n-1}$$

$$= \sum_{n=1}^{N-2} \frac{1}{n} - \sum_{n=2}^{N-1} \frac{1}{n}$$

$$= \left(1 + \sum_{n=2}^{N-2} \frac{1}{n} \right) - \left(\sum_{n=2}^{N-2} \frac{1}{n} + \frac{1}{N-1} \right) = 1 - \frac{1}{N-1}$$

$$S = \sum_{n=3}^{\infty} \frac{1}{n^2 - 3n + 2} = \lim_{N \rightarrow +\infty} S_N = 1$$