

ATS 2019 (MANGANE Stücken)

$$(1) \quad \underline{I} = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{1 + \cos^2 x} dx$$

Méthode des invariants: $\frac{\sin^3(-x) d(-x)}{1 + \cos^2(-x)} = \frac{\sin^3 x dx}{1 + \cos^2 x}$ d'après $X = \cos x$

$$dX = -\sin x dx \quad \underline{I} = \int_1^0 \frac{\sin^3 x}{1 + X^2} \left(-\frac{dX}{\sin x} \right) = \int_0^1 \frac{\sin^2 x}{1 + X^2} dX$$

$$\underline{I} = \int_0^1 \frac{1 - \cos^2 x}{1 + X^2} dx = \int_0^1 \frac{1 - X^2}{1 + X^2} dX = \int_0^1 \left(1 + \frac{2}{1 + X^2} \right) dX$$

$$= \left[-X + 2 \arctan(X) \right]_0^1 = -1 + 2 \arctan 1 = \frac{\pi}{2} - 1 = \frac{\pi - 2}{2}$$