

A 2019 (Lacroix Thomas).

① On pose  $X = Z^2 - 8Z$

$$X^2 + 40(Z^2 - 8Z) + 375 = 0 \quad \Delta = 40^2 - 4 \cdot 375 = 1600 - 1500 = 100 = 10^2$$

$$X = \frac{-40 \pm 10}{2} = \begin{cases} -25 \\ -15 \end{cases}$$

Premier cas:  $Z^2 - 8Z = -25 \quad Z^2 - 8Z + 25 = 0 \quad \Delta = 64 - 100 = -36 = (6i)^2$

$$Z = \frac{8 \pm 6i}{2} = \underline{4 \pm 3i}$$

Second cas:  $Z^2 - 8Z = -15 \quad Z^2 - 8Z + 15 = 0 \quad \Delta = 64 - 60 = 4$

$$Z = \frac{8 \pm 2}{2} = \begin{cases} 5 \\ 3 \end{cases} \quad \text{4 solutions: } \underline{3, 5 \text{ et } 4 \pm 3i}$$

②  $I = \int_0^1 x^3 e^{-x/2} dx$  P&D  $\begin{cases} u = x^3 & u' = 3x^2 \\ v' = e^{-x/2} & v = -2e^{-x/2} \end{cases}$

$I = [-2x^3 e^{-x/2}]_0^1 + 6 \int_0^1 x^2 e^{-x/2} dx$  P&P  $\begin{cases} u = x^2 & u' = 2x \\ v' = e^{-x/2} & v = -2e^{-x/2} \end{cases}$

$I = -2e^{-1/2} - 12 [2x e^{-x/2}]_0^1 + 24 \int_0^1 x e^{-x/2} dx$  P&D  $\begin{cases} u = x & u' = 1 \\ v' = e^{-x/2} & v = -2e^{-x/2} \end{cases}$

$I = -14e^{-1/2} - 48 [2x e^{-x/2}]_0^1 + 48 \int_0^1 e^{-x/2} dx$

$= -14e^{-1/2} - 48e^{-1/2} - 96 [e^{-x/2}]_0^1 = -62e^{-1/2} - 96e^{-1/2} + 96 = \boxed{96 - 158e^{-1/2}}$

③  $\frac{1}{2+x} = \frac{1}{2} \left( \frac{1}{1+x/2} \right) = \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \left( \frac{x}{2} \right)^n \quad \forall x \in ]-1, 1[ \quad \text{car } \forall x \in ]-2, 2[$

donc  $\frac{1}{2+x} = \sum_{n=0}^{+\infty} \frac{(-1)^n}{2^{n+1}} x^n, \quad \text{pour } |x| < 2$