

ATS 2019 (COLLE Simon)

③ $f(x,y) = \operatorname{arctan}\left(\frac{x+y}{1-xy}\right)$

$\operatorname{arctan}(u)' = \frac{1}{1+u^2} \cdot u'$

$\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{x+y}{1-xy}\right)^2} \cdot \frac{(1-xy) + y(x+y)}{(1-xy)^2} = \frac{1+y^2}{(1-xy)^2 + (x+y)^2} = \frac{1+y^2}{1+x^2y^2+x^2+y^2}$

$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{x+y}{1-xy}\right)^2} \cdot \frac{(1-xy) + x(x+y)}{(1-xy)^2} = \frac{1+x^2}{1+x^2y^2+x^2+y^2}$

$g(x,y) = y^x \quad \frac{\partial}{\partial y} = xy^{x-1}$

$\frac{\partial}{\partial x} = \frac{\partial}{\partial x}(e^{x \ln y}) = \ln y e^{x \ln y}$

⑤ voir ATS 2019 (TRAVEL)

① $\det A = \begin{vmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{vmatrix} \begin{matrix} \text{Col 1} \\ \text{Col 2} \\ \text{Col 3} \end{matrix} = -1 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1(3-4) = 1$

$\det A \neq 0$, donc A est inversible.

Dans A , on a $\vec{u}, \vec{v}, \vec{w}$ en fonction de $\vec{e}_1, \vec{e}_2, \vec{e}_3$. Dans A^{-1} , c'est l'inverse.

$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{pmatrix} \begin{matrix} \vec{u} \\ \vec{v} \\ \vec{w} \end{matrix} \rightarrow \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \begin{matrix} \vec{u} \\ \vec{v} \\ \vec{w} \end{matrix}$
 $\vec{u} = \vec{e}_2 \Rightarrow \vec{e}_2 = \vec{u}$. On vérifie. ok
 $\vec{w} - \vec{v} = \vec{e}_3$. On vérifie. ok

colonne 3: $\vec{e}_1 + \vec{e}_2 + 3\vec{e}_3 = \vec{w} \Rightarrow \vec{e}_1 = \vec{w} - \vec{e}_2 - 3(\vec{w} - \vec{v})$
 $= 4\vec{w} - 2\vec{e}_2 - 6\vec{v} \Rightarrow \vec{e}_1 = -\vec{u} - 3\vec{v} + 2\vec{w}$

On vérifie ok

$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
 VERIF

Donc $A^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ -3 & 0 & 2 \\ 2 & 0 & -1 \end{pmatrix}$