

ATS 2019 (ALIXANT C)

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

① $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ et $B = A - I$ $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $B^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

et $B^3 = B^2 \cdot B = \dots = 0$, donc $B^n = 0, \forall n \geq 3$

$$A^n = (I+B)^n = \sum_{k=0}^n \binom{n}{k} B^k I^{n-k}$$

On peut utiliser la formule du binôme car B et I commutent

$$A^n = B^0 + n B^1 + \frac{n(n-1)}{2} B^2 = I + n \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + \frac{n(n-1)}{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & n & \frac{n(n-1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

② $\int_{-1}^2 x|x| dx = \int_{-1}^0 + \int_0^2 = \int_{-1}^0 -x^2 dx + \int_0^2 x^2 dx$

car ici $|x| = -x$

↑

car ici $|x| = x$

$$= \left[-\frac{x^3}{3} \right]_{-1}^0 + \left[\frac{x^3}{3} \right]_0^2 = -\frac{1}{3} + \frac{8}{3} = \frac{7}{3}$$

③ $\ln\left(\frac{n+1}{n-1}\right) = \ln\left(\frac{n(1+\frac{1}{n})}{n(1-\frac{1}{n})}\right) = \ln\left(1+\frac{1}{n}\right) - \ln\left(1-\frac{1}{n}\right)$

$$= \frac{1}{n} + o\left(\frac{1}{n}\right) - \left(-\frac{1}{n} + o\left(\frac{1}{n}\right)\right) = \frac{2}{n} + o\left(\frac{1}{n}\right) \xrightarrow{n \rightarrow +\infty} 0$$