

2018 (SAV (Baphiste))

$$(2) \sqrt{1+x} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}(-\frac{1}{2})x^2}{2!} + \mathcal{O}(x^2) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \mathcal{O}(x^2)$$

$$\sqrt{1+2x} = 1 + \frac{1}{2}(2x) - \frac{1}{8}(2x)^2 + \mathcal{O}(x^2) = 1 + x - \frac{1}{2}x^2 + \mathcal{O}(x^2)$$

$$\begin{array}{r} 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \mathcal{O}(x^2) \\ - (1 + x - \frac{1}{2}x^2 + \mathcal{O}(x^2)) \\ \hline -\frac{1}{2}x + \frac{3}{8}x^2 + \mathcal{O}(x^2) \\ - (-\frac{1}{2}x - \frac{1}{2}x^2 + \mathcal{O}(x^2)) \\ \hline \frac{7}{8}x^2 + \mathcal{O}(x^2) \end{array}$$

$$\text{Donc } \sqrt{\frac{1+x}{1+2x}} = 1 - \frac{1}{2}x + \frac{7}{8}x^2 + \mathcal{O}(x^2)$$

$$\text{Ln} \left(\sqrt{\frac{1+x}{1+2x}} \right) = \text{Ln}(1+x) \text{ avec } x = -\frac{1}{2}x + \frac{7}{8}x^2 + \mathcal{O}(x^2)$$
$$= x - \frac{x^2}{2} + \mathcal{O}(x^2)$$

$$= -\frac{1}{2}x + \frac{7}{8}x^2 - \frac{1}{2} \left(\frac{1}{4}x^2 \right) + \mathcal{O}(x^2)$$

$$= -\frac{1}{2}x + \frac{3}{4}x^2 + \mathcal{O}(x^2)$$